HISTORICAL ESSAY

ON THE

RISE AND EARLY PROGRESS

OF THE

DOCTRINE OF LIFE-CONTINGENCIES

IN ENGLAND,

LEADING TO THE ESTABLISHMENT OF THE FIRST

LIFE-ASSURANCE SOCIETY IN WHICH

AGES WERE DISTINGUISHED.

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PREFACE.

ALTHOUGH the importance of the Doctrine of Life-Contingencies as the basis of Life-Assurance and Annuity Transactions, is universally acknowledged, yet, a separate Treatise on the HISTORY of the subject, does not appear to have been hitherto published.

The present Essay represents an attempt to partly obviate this deficiency, and to thus pourtray to the general reader the early developement of a doctrine, in whose trustworthiness so many hundreds of the community have since acquired a pecuniary interest.

In endeavouring to fulfil this object, the writer has sought rather to contract, than to extend the relative details, having necessarily felt doubtful in the absence of similar publications, whether the topic, in its historical form, would be deemed of sufficient interest, to warrant in the first instance, the production of a more elaborate treatise. It is hoped, however, that by the aid of the Chronological arrange-

ment adopted, and the frequent employment of illustrative quotation, that the general reader will experience but little difficulty in tracing collateral sources of amplification, should he be hereafter tempted to prolong his inquiry, into the more detailed character of this peculiar branch of human knowledge.

The Doctrine of Life-Contingencies being also constructively a doctrine of calculation, the writer has framed, as a species of counterpart to the verbal portion of the present Essay, a brief Chronological Arrangement of the relative formulæ.

HISTORICAL ESSAY ON THE RISE AND EARLY PRO-GRESS OF THE DOCTRINE OF LIFE - CONTIN-GENCIES IN ENGLAND.

DURING the early periods of the history of calcula-tive subjects, peculiar fitness has generally been ascribed to particular numbers. Among these the number *seven* has commonly been allotted a promi-nent rank, the primary attempts at Life-measurement constituting an additional instance of the numerical favouritism;—

The number seven appears to have been first applied to the estimation of life, in the popular notion, of human existence being naturally divisible into septennial periods, and consequently every seventh year critical as to survivance.

The origin of this supposition, is probably to be referred to the frequent occurrence of expressions of a septenary character in Holy Writ. The fifteenth chapter of Deuteronomy will be found indeed wholly devoted to inculcate, that "the Seventh year is a year of release;" and also, throughout the Scriptures, numerous other references, fully warranting in a religious sense, this implied fitness of Septennial division.

The notion of Seven-year epochs from birth, seems gradually to have been considered as applicable to any period of life, and eventually to have been extended to Financial matters, upon the presumption that pecuniary Life-benefits beyond Seven years realization, were too precarious to be worthy of pre-arranged valuation. A single life-income, was thus incidentally adjudged as reciprocal to seven years' duration, two Lives to fourteen, and three Lives to twenty-one; and thus the well-known arrangement of Leases, for One, Two, and Three Lives, and for Seven, Fourteen, and Twenty-one years, appears to have arisen, and to have been considered as affording commensurate limits.*

This simple apportionment seems to have been so consistent with popular notions, as to be ultimate-

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* Probably the concurrent distinctions of Legal Ages may have lent aid to the above relations. Thus, in Magna Charta, (Henry 3, Feb. 9, 1224,) we find full age *(plene etatis)* described as Twenty-one years, " *cum ad etatem pervenerit scilicet viginti et unius anni habeat* " *hereditatem suam sine relevio et sine fine."* (Rot^o. Turri Inspex: Ed. I.—Bracton (1240-1260) tells us that the age or time for betrothment or early dowry *("maritagium" " aid ' pur file marier,' ")* was seven years. (De Leg. Lib. II),—Littleton, in his Tenures, (1474-80) says, " *lage de discretion est dit lage de 14 ans."* (Lib. II. sec. 104.) Thus, Seven, Fourteen, and Twenty-one years, were all legally-defined periods.

It is worthy of notice that in the original Magna Charta, granted by King John, (June 15, 1215,) the phrase is merely " *cum ad etatem* " *pervenerit habeat hereditatem suam sine relevio et sine fine."* (Cotton M. S. *Pine sc.*) but after John's death, his son, Henry III, having applied to the Pope (Honorius III,) to be declared of full age at seventeen years, and to be allowed to realize the royal possessions and prerogatives, (Matt. Paris Hist. Maj.) the Barons, on his confirming the Charter of his father, appear to have thought it necessary to prevent general misconstruction, to insert the words " *scilicet viginti et unius anni."*

ly thought suitable for authoritative adoption, a statute being passed in 1540 known as the enabling statute for granting Leases against the sudden action of entail, in which such Leases were not however " to bee made aboue the number of twentie and one years, or three " lives." (32 H. 8, cap. 28, s. 2).

" And what hath been said concerning a lease for three lives, doth " hold for a lease for one and twentie years." (Sir Ed. Coke, First Inst. " chap. 7, s. 58).

When it is added, that the current value of such Leases was assumed at seven years' purchase, the reader may form a tolerable notion of the estimates anciently adopted.*

It is unnecessary to further trace this species of assessment here, but as leases furnished for many years, the only demand for Life-valuations, the reader's attention has been directed to these particulars, in elucidation of arrangements long afterwards subsisting. The gradual ascertainment

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* Seven years' purchase for a lease of twenty-one years duration was virtually allowing £13 4 8½.08 ⅌ cent. interest, ($\varphi (1 + r) =$ $(1 + r)^{22} - (1 + \frac{1}{7}) (1 + r)^{21} + \frac{1}{7}$ whence $(1 + r)$ the first term of the roots $= 1.132355$.)

The current value exemplified by seven years' purchase, probably formed a collateral guide to the first legal recognizance and estimate of interest. Thus Œcroid's lease tables, generally referred to Henry the Eighth's reign, in connection with the dissolution of the monasteries, (31 H. 8, cap. 13), were first computed in relation to the " seven year value;" but as a portion of the rent in future leases on the Abbey lands was to be reserved to the king's use, seven years' purchase of the full rent reduced the commensurate interest to about 11 ⅌ cent. (11.1764), until 1545, when a larger rent was reserved, and interest first legally declared at 10 ⅌ cent. ⅌ annum, " the " summe of tennepounds in the hundred for one whole yeare, and so " after the rate abouesaid for a more or lesser summe or for a longer " or shorter time." (37 H. 8, cap. 9, see. 4.)

of the number of deaths among the general community, will furnish us with the next and more important source of inquiry.

Although individual deaths, in a country so strongly enforcing the laws of inheritance, must have continually exemplified the importance of such events being recorded; yet general registration seems to have been disregarded in England until about the thirtieth year of Henry the Eighth's reign, and to have been instituted, merely as a part, of certain regulations relating to ecclesiastical matters.

On the Papal authority being openly contemned and Henry the Eighth declared " Supreme Head of the Church," to " extirp all Errors, Heresies, and other Enormities," (26 H. 8, cap. 1), he created Thomas Cromwell, Vicar General, and intrusted him (Oct. 1535) with a Commission for a general Visitation of Religious Establishments.

[1538] The result of this Visitation was published, and afforded not only the desired pretext for abolishing the Monasteries (31 H. 8, cap. 13), but also convenient occasion for the issual of new Protestant Rules and Regulations for the kingdom generally. Among these was an Injunction made, Sept. 1538, by Cromwell, to the effect, that every " Parson, Vicar, or Curate," was to keep a true and faithful account of all " Weddings, Christenings, and Funerals," in his particular Parish, subject to a penalty of three shillings and four pence for every omission, (*vide* BishopBurnet'sHistory of theReformation,vol.I.Appendix,p.170)

That this visitation of Lord Cromwell (1535-38)

was the origin of general registration of deaths, is not only probable in itself, from the want of anterior evidence, but is also confirmed by the curious fact, that though the industrious Dr. Short, in 1740-50 collected registers of " Christenings and Burials " of one hundred and sixty Country Parishes, besides many Chapels in " different and distant Parts of England, lying in sundry Situations, " on various Soils, and the Inhabitants have different Businesses and " Ways of Life." (*vide* Short's New Observations, 1750, page 13) ; yet his treatise does not present any example of registers precedent to 1535-38, although he appears to have obtained several commencing from that period, the parishes of Worksop, in Nottinghamshire, and Melton-on-Hill, Yorkshire, affording instances.

[1547] The registration thus established by Cromwell's Injunction does not seem to have attracted further notice until 1547 (1, Edw. VI.) when a new Visitation and Injunction were ordered, and an attempt made, though finally abandoned, to frame a statute for establishing a " Treasure House" in every county, for the keeping of such records, (apud Parl. Rep. 669, p.p. 33-74).

[1554] During the sway of Queen Mary, the Papal supremacy having been re-established, the former system of registration, from its depending upon beneficed clergymen, probably became disused, for, in 1554, the Roman Catholic Bishops, in their Visitations, were directed to ascertain whether each " Parish Priest" kept a register " of the Sponsors, of the Married, and of the Dead." (*ibid.* p. 33).

[1558] On the succession of Elizabeth, in 1558, the Injunction, respecting each " Parson, Vicar, or Curate," was reissued, and all precedent registers

ordered to be copied into parchment books. (*vide* Iniunctions geven by the Quenes Maiestic, 4to. 1559.)

[1562] In pursuance of these successive Orders of different sovereigns, registration of deaths appears to have been thenceforth more or less practised by the country generally;* but the Metropolis, either by its local privileges, or other causes, seems to have totally evaded the usual ordinances, and it was not until the subject of mortality acquired a fearful interest by the visitation of the Plague, in 1562, that any account of London burials was instituted.

Maitland, in his History of London, says, (Ex bibl. Sir Hans Sloane.) " In the year 1562, a grievous Pestilence raged " in this City, therefore, in order to know the Increase and Decrease " of the same, 'twas judg'd necessary to take an Account of the Num- " ber of burials, which being the First of the Kind that was ever " taken in London, it commenced on the 1st of January, Anno. 1562, " and ended the last of December, 1563, whereby it appears that " the Number Total buried within the City and Suburbs in that " Year, amounted to 23,630, whereof of the Plague 20,136." (Maitland's History of London, p. 736.)† The plague subsiding, the registration of Metropolitan burials was discontinued.

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* The subsequent changes in Parish Registers, after the Injunction of Elizabeth, were principally of a legal character, with a view to their improvement as documentary evidence. The reader will find ample details of such enactments, &c. in Mr. J. S. Burn's Treatise on the subject.

† It may be well to remark, that at the time alluded to, the Catholic, or old style, was in use. In accordance with such, the day of the Annunciation to the Virgin Mary, or Lady-day, (March 25th,) was reckoned as the first day of the year; consequently in this sense, the above-mentioned period, from January 1, 1562, to December 31, 1563, would but comprise one year, although, at first perusal, two might seem to be implied. The new or present style was not adopted in England, until January 1, 1752. (24 Geo. II. cap. 23.)

[1592] In 1592, the plague again appearing in London, the Fraternity of St. Nicholas, afterwards called the Company of Parish Clerks, instituted and provided a weekly account of the burials, which, at the end of the year, (Dec. 21, 1592, to Dec. 21, 1593,) were announced as amounting to 17,844, " whereof have died of the Plague, 10,662; Christen'd this Year, " 4,021; Parishes clear of the Plague, none." (Parish Clerks' Register, *ibid.*)

The plague still continuing, (1594) weekly accounts of burials were issued to Dec. 18th, 1595, when the practice was again abandoned, the plague having subsided in the course of the year.

[1603] In 1603 the plague re-appearing, the Bills of mortality were resumed, and were thenceforth continued uninterruptedly, without being merely dependent upon plague visitations; the Fraternity of St. Nicholas, who had hitherto undertaken the registration, ultimately procuring, in 1611, a charter from James I, to incorporate them as the Company of Parish Clerks, and were formally intrusted with the future regulation and issue of the " London Bills," (Maitland, pp. 737-1249).

[1625] In 1625, the plague becoming unusually prevalent, the Company obtained permission to erect a printing press in their hall, for better dispatch in satisfying public curiosity, as to the places chiefly infected; and accordingly the Bills, in lieu of merely denoting the total number of deaths, also specified the number appertaining to each parish, the practice being afterwards continued from year to year.

[1629] In 1629, an account of the diseases and casualties was for the first time added; for though

such details had been returned to the Company
since 1604, yet it probably appeared doubtful whether
such distinctions would be generally tolerated, and
accordingly, on their first promulgation in this year,
(1629) it was deemed politic to issue two sets of
Bills, one with, the other without, such additions.
(Parish Clerks' Register, *ibid*.)

The blank duplicature was afterwards disconti-
nued, and the weekly bills, thus stating the number
of burials, (males and females,) christenings, and
nature of diseases, without, however, any distinction
as to ages, continued unaltered for many years, and
were but little noticed, unless the plague or other cir-
cumstance directed public attention to the subject.

[1662] In 1662 was published the first book on
Life-contingencies as a topic worthy of distinct com-
ment, and will thus indirectly afford us a relative'
opportunity of becoming acquainted with the cur-
rent notions characterizing those times.

The work was entitled, " Natural and Political Observa-
" tions, mentioned in a following Index, and made upon the Bills of
" Mortality, &c. by John Graunt, Citizen of London."

The author thus states his own purpose, " Having
" been born and bred in the City of London, and having always ob-
" served that most of them who constantly took in the weekly Bills of
" Mortality, made little other use of them, than to look at the foot
" how the Burials increased or decreased, and among the Casualties
" what had happened rare and extraordinary in the week current; so
" as they might take the same as a Text to talk upon in the next Com-
" pany, and withal in the Plague time, how the Sickness increased or
" decreased, that the Rich might judg of the necessity of their remo-
" val, and Tradesmen might conjecture what doings they were like to
" have in their respective dealings;
  " Now I thought that the Wisdom of our City had certainly design-

" ed the laudable practice of taking and distributing these Accompts for
" other and greater uses than those above-mentioned, or, at least, that
" some other uses might be made of them; and thereupon I casting
" mine eye upon so many of the general Bills as next came to hand, I
" found encouragement from them to look out all the Bills I could, and
" (to be short) to furnish myself with as much matter of that kind,
" even as the hall of the Parish Clarks could afford me; the which
" when I had reduced into tables, (the Copies whereof are here insert-
" ed), so as to have a view of the whole together, in order to the more
" ready comparing of one Year, Season, Parish, or other Division of
" the City, with another in respect of all Burials and Christnings, and
" of all the Diseases and Casualties happening in each of them respec-
" tively; I did then begin not onely to examine the Conceits, Opinions,
" and Conjectures, which upon view of a few scattered Bills I had
" taken up, but did also admit new ones, as I found reason and occa-
" sion from my Tables.

" Moreover, finding some Truths and not-commonly-believed
" Opinions to arise from my Meditations upon these neglected Papers,
" I proceeded further to consider what benefit the knowledge of the
" same would bring to the World, that I might not engage myself in
" idle and useless Speculations." (Graunt's Obs. 1662, Pref.)

Mr. Graunt then details the progressive history of the Bills; such information however, having been already supplied to the reader from the Parish Clerks' Register, need not be here further commented on.

He then adds: " We have hitherto described the several
" steps whereby the Bills of Mortality are come to their present state;
" we come next to show how they are made and composed, which is
" in this manner, viz: When any one dies, then, either by tolling or
" ringing of a bell, or by bespeaking of a grave of the sexton, the same
" is known to the searchers corresponding with the said sexton. The
" searchers hereupon (who are ancient matrons sworn to their office)
" repair to the place were the dead corpse lies, and by view of the
" same, and by other inquiries, they examine by what disease or casu-
" alty the corpse died. Hereupon they make their report to the parish
" clerk: and he, every Tuesday night, carries in an accompt of all the
" burials and christenings happening that week, to the clerk of the
" Hall. On Wednesday, the general accompt is made up, and print-

" ed; and on Thursday published, and dispersed to the several families
" who will pay four shillings per annum for them." (*ibid.* chap. 1.) *

Mr. Graunt subsequently discusses, *seriatim*, in several Chapters, the contents of the Bills, and, considering the date at which he wrote, with considerable skill; with respect to such details we necessarily refer the reader to the treatise itself, contenting ourselves with a citation from Chapter 11, as a specimen of the methods of reasoning adopted in those days from the very limited statistical data to which Mr. Graunt and his cotemporaries had access.

" Of the Number of Inhabitants. I have been several times in
" company with Men of great Experience in this City, and have heard
" them talke seldome under Millions of People to be in London; all of
" which I was apt enough to believe, until on a certain day one of
" eminent Reputation was upon occasion asserting, That there was, in
" the year 1661, two Millions of People more than Anno 1625 before
" the great Plague. I must confess, that until this provocation, I had
" been frighted with that misunderstood Example of David, from at-
" tempting any computation of the People of this populous place;
" but hereupon, I both examined the lawfulness of making such inqui-
" ries, and being satisfied thereof, went about the work it self in this
" manner, viz.

" First, I imagined, That if the Conjecture of the worthy Person
" afore-mentioned had any truth in it, there must needs be about six
" or seven Millions of People in London now; but repairing to my
" Bills, I found that not above 15,000 per Annum were buried, and,

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* However antiquated this method of proceeding may appear, yet the London Bills of Mortality, the only publicly-issued data on the subject in England, continued very nearly unchanged in their manner of collection, until the period of their reconstruction in 1836 (6 & 7 W. 4. cap. 85), when a General Register Office was established. It is right to add, that the well-considered foundation of this establishment, and the skill of its officers, have already enabled England to boast of perhaps more complete data, as to the general rate of mortality than any other nation possesses, although nearly similar establishments had existed on the Continent, in Sweden, and in America, for many years previously.

" consequently, that not above one in four hundred must die per An-
" num, if the Total were but six Millions. *　*　*　* I there-
" upon endeavoured to get a little nearer thus, viz.—

" I considered that the number of Child-bearing Women might be
" about double to the Births: forasmuch as such Women one with
" another have scarce more than one Child in two years. The number
" of Births I found by those years wherein the Registries were well
" kept, to have been somewhat less than the Burials. The Burials in
" these late years at a Medium, are about 13,000, and, consequently,
" the Christenings not above 12,000. I therefore esteemed the num-
" ber of Teeming women to be about 24,000; then I imagined that
" there might be twice as many Families as of such Women; for that
" there might be twice as many Women aged between 16 and 76 as
" between 16 and 40, or between 20 and 44, and that there were
" about eight Persons in a Family, one with another, viz.—the Man
" and his Wife, three Children, and three Servants or Lodgers; now
" 8 times 48,000 makes 384,000." 　*　*　*　*

" We have found that of 100 quick Conceptions, about 36 of
" them die before they be six years old, and that perhaps but one
" surviveth 76; we having seven Decads between six and 67, we
" sought six mean proportional numbers between 64, the remainder,
" living at six years, and the one, which survives 76, and find that the
" numbers following are practically near enough to the truth, for men
" do not die in exact proportions, nor in Fractions, from whence arises
" this Table following:—

Viz. Of one hundred there dies within the first six years	36	The third dead . . 9
The next ten years or decad	24	The fourth 6
		The next 4
The 2nd dead . . .	15	The next 3
		The next 2
		The next 1

" From whence it follows that of the said 100 conceived, there re-
" main alive at six years' end 64.

" At sixteen years' end .	40	At fifty-six 6
At twenty-six . . .	25	At sixty 3
At thirty-six . . .	16	At seventy-six . . . 1
At forty-six	10	At eighty . . . . 0"*

(Graunt's Observ. Chap. 11.)

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* The reader conversant with such matters will detect in Mr.
Graunt's arrangement the first semblance of a Table of Mortality.

This treatise of Mr. Graunt's, the first, and considering its date, one of the best on the subject, seems to have been properly appreciated, and the author encouraged: for in Bishop Sprat's cotemporary History of the Royal Society, we find the following illustration of the eligibility of men of "all conditions" to become Members. "I will shew by one instance; and it is "the Recommendation which the King himself was pleased to make "of the judicious Author of the 'Observations on the Bills of Mor-"'tality,' in whose Election it was so far from being a Prejudice that "he was a Shop-keeper of London, that his Majesty gave this parti-"cular Charge to his (*i. e.* the Royal) Society, that if they found any "more such Tradesmen, they should be sure to admit them all without "more ado." (Bishop Sprat's History of the Royal Society, p. 67.) Mr. Graunt, thus patronized, was enabled to publish a second edition in the same year.

[1665] In 1665, the work was printed by order of the Royal Society, and a "Third Edition, much Enlarged," accordingly issued, though in a more convenient compass, the size of the book being changed from 4to to 8vo.* In the same year, the "Fourth Impression" appeared, but was printed at Oxford, the plague having become so prevalent, during the latter part of this year, in the Metropolis, as to leave the City almost deserted, the public companies retiring to Oxford. "The sickness in 1665, which "raged in most parts of the kingdom, never visited any person there "although the terms were there kept, and the Court and both Houses "of Parliament did there reside." (*vide* Quincy's Essay on Pestl. Diseases, p. 8.; Dr. Plot's History of Oxfordshire, chap. II.; and Dr. Birch's History of Royal Society, vol. II. p. 62.)†

This circumstance of the plague occurring during

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* From this Edition, we have selected the preceding quotations.

† During "the great plague-year," (1665) several publications commenting on the Bills of Mortality were issued, as "Bell's Lon-

the earlier years of Graunt's publication, may, in part, account for the rather extraordinary demand for the work, (probably as a matter of curiosity as to the Bills of Mortality it contained,) for the Great Fire of London happening in 1666, the year following the fourth edition, and by extensive devastation rendering the City less crowded with buildings, the plague finally disappeared, and ten years elapsed before a new edition of Graunt's treatise was called for, a fifth edition being then published, (1676), under the superintendence of his marriage relation, Sir William Petty; Mr. Graunt having died two years previously.

[1674] Sir W. Petty appears thenceforth to have more or less devoted attention to the subject, as exemplified by his "Essay on Political Arithmetics concerning " the People, Housings, &c. of London and Paris;" " Essay concern- " ing the Multiplication of Mankind, and the Growth of the City of " London;" " Observations on the Dublin Bills of Mortality, &c."

These, however, being chiefly discursive, in illustration of maxims, &c. in Political Economy, require no further mention in the present place; but in a " Discourse on Duplicate Proportion," read before the Royal Society, and published separately in 1674, he advances the following rule, which appears to constitute the first attempt to define the comparative rates of mortality at different ages.

He instances 19 uses of Duplicate Proportion, and assigns " The Eleventh Instance. In the Life of Man and its

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don's Remembrancer, or a true Account, &c."—" The Four great Plagues, &c."—" Reflections on the Weekly Bills of Mortality."— *vide* also the History " Λοιμωλογια *sive pestis*, &c.," (1672) by Dr. Hodges, one of the four physicians appointed by Charles II. " to attend the infected on all occasions."

" DURATION. It is found by Experience that there are more persons
" living of between 16 and 26 years old, than of any other Age or
" Decade of years in the whole life of Man, (which David and Expe-
" rience say to be between 70 and 80 years). The reasons whereof are
" not abstruse, viz. because those of 16 have passed the danger of
" Teeth, Convulsions, Worms, Rickets, Measles, and Small Pox, for
" the most part; And for that those of 26 are scarce come to the Gout,
" Stone, Dropsie, Palsies, Lethargies, Apoplexies, and other Infirmities
" of Old Age. Nor whether these be sufficient reasons, is not the pre-
" sent Enquiry, but taking the afore-mentioned assertion to be true, I
" say that the Roots of every number of Men's Ages under 16, (whose
" Root is 4) compared with the said number 4, doth shew the propor-
" tion of the likelihood of such men's reaching 70 years of age. As,
" for example: 'Tis 4 times more likely that one of 16 years' old
" should live to 70, than a new-born Babe; 'tis 3 times more likely
" that one of 9 years old should attain the said age of 70, than the said
" Infant. * * * And so forward, according to the Roots
" of any other year of the declining Age, compared with a number
" between 4 and 5, which is the Root of 21, the most hopeful year for
" Longœvity, as the mean between 16 and 26, and is the year of per-
" fection according to the sense of Our Law, and the Age for whose
" life a Lease is most valuable. To prove all which I can produce the
" accompts of every Man, Women, and Child, within a certain Parish
" of above 330 Souls, all of which particular Ages being cast up, and
" added together, and the Sum divided by the whole number of Souls,
" made the quotient between 15 and 16, which I call (if it be Constant
" or Uniform) the Age of that Parish, or numerous Index of Longœvity
" there. Many of which Indexes for several times and places would
" make an useful Scale of Salubrity for those places, and a better Judg
" of Ayres than the conjectural Notions we commonly read and talk
" of. And such a Scale the King might as easily make for all his Do-
" minions as I did this for this one Parish." (Sir W. Petty's Discourse
on Dup. Prop. 1674. p. 82.)

During the succeeding ten years, 1674 to 1684, the
subject of life-valuation seems to have been chiefly
considered with respect to Leases; this subject there-
fore again requires an occasional notice.

The restricted limit of the term to " twenty-one
" years, or three lives," declared in Henry the Eighth's

reign, as to mortgages (32 H. 8, cap. 28), (*vide* p. 7) had been further stringently applied to College and Ecclesiastical leases during the reign of Elizabeth, " And for that long and unreasonable Leases made by Colledges " *   *   be the chiefest causes of the Delapidations   *   *   im- " poverishing of all successors   *   *   *   be it enacted—all leases " *   *   *   to bee made by any master and fellowes of any Col- " ledge   *   *   other than for terme of one and twentie yeares, " or three lives, shal be utterly void." (13 El. cap. 1, s. 3.)

This system of leasing not only continued in such matters, but remained subject to the old methods of valuation, for as in Œcroid's original tables when first compiled, the fine for seven years, or one life lapsed, was, after allowing for reserved rent, " one and one fifteenth" years' purchase, so by common parlance this had become simply " one year to renew seven." In order, therefore, to charge fines for other terms, consistently with the rate already fixed by usage for seven years, it became a desideratum to have the relative interest thus involved correctly set forth.

[1683-6] Apparently to facilitate this, in 1683-6 a set of tables were published at Cambridge, for " Renewing and Purchasing of the Leases of Cathedral Churches and " Colleges, &c. also Tables for renewing and Purchasing of lives, &c." The principle of construction adopted in these Tables, was to develope what exact rate of interest was implied when one year's purchase was the fine for renewal, other rates being incidentally added for ordinary leases. Sir Isaac Newton being then at Cambridge, (Lucasian Professor, *vice* Barrow), it appears to have been thought politic to obtain his sanction, or " *imprimatur*," as to the correctness of the tables, the following sentence in connection with his name being added to the title-

page: " *Methodus hujus libri rectè se habet numerique ut ex qui-* " *busdam ad calculum revocatis, judico, satis exactè computantur.*"
Upon this slender claim, the Tables at a subsequent period were circulated as a work of Sir Isaac Newton's, " Sir Isaac Newton's Tables," by which title they are still known, although Sir Isaac's original cognizance of the work appears to have been to merely confirm the *(*Q. E. I.*)* correctness of a single Table, relative to the established usage of renewing college leases.

The publication (disregarding its extraneous pretensions) affords, in connection with its date, the following curious indication of the conjectural methods then commonly adopted in Life valuations, " The way of purchasing by lives, was commonly to reckon " one life as a lease of seven years, two lives as a lease of fourteen " years, and three lives as a lease of twenty-one years. But this way " seeming unequal, there is another way, which is more agreeable to " reason, and it is this, viz. for every life to decrease one year; as if " one life be reckoned as a lease for ten years, then two will be as a " lease for nineteen, and three as a lease for twenty-seven years, &c.;

" So if you reckon one life as a lease of nine years, then two will " be as a lease of seventeen, three as a lease of twenty-four, &c.

" So if one single life be reckoned as a lease of twelve years, then two " will be as a lease of twenty-three, three as a lease of thirty-three years, " &c."

After some further exemplifications, a Table is added of the following form, which appears to constitute the first species of Life Annuity Table offered for public use:—

A Table for the Purchasing of Lives.

Number of Lives.	Number of Years.	What they are worth at 7l. p. ct.				Number of Lives.	Number of Years.	What they are worth at *7l. p. ct.				Number of Lives.	Number of Years.	What they are worth at 6l. p. ct.			
		Years.	Quarters.	Months.	Decimal Parts.			Years.	Quarters.	Months.	Decimal Parts.			Years.	Quarters.	Months.	Decimal Parts.
1	10	7	0	0	3	1	9	6	2	0	2	1	12	8	1	1	6
2	19	10	1	1	0	2	17	9	3	0	1	2	23	12	1	0	6
3	27	11	3	2	6	3	24	11	1	2	6	3	33	14	1	2	6
4	34	12	3	1	1	4	30	12	1	1	8	4	42	15	0	2	7
5	40	13	1	0	9	5	35	12	3	2	2	5	50	15	2	2	7
6	45	13	2	1	2	6	39	13	1	0	1	6	57	16	0	0	8
7	49	13	3	0	2	7	42	13	1	2	4	7	63	16	0	2	8
8	52	13	3	1	3	8	44	13	2	0	6	8	68	16	1	1	2
9	54	13	3	1	6	9	45	13	2	1	2	9	72	16	1	2	0

("Tables for renewing of Leases, &c. and Purchasing of Lives,"
1683-86, chap. 6.)

[1692.] However imaginary the values in the above Table may appear, yet, in 1692, an Act of Parliament was passed, in which it was stated, after a provision for raising money by a Tontine, that parties " shall have and receive, for every sum of one hundred pounds by " them respectively advanced and paid, a yearly annuity, rent, or " payment, of fourteen pounds of lawful English money (and pro- " portionately for a greater sum), and for and during the life of such

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* The repetition of 7 per cent. may, on first perusal, appear merely a misprint, but on considering the fanciful construction of the Table, the cause of the duplicature will become apparent. In the first division one Life is assumed as equal in duration to 10 years, in the second, 9 years are assigned to it, consequently the two divisions, though in connection with similar rates of interest, represent different values. It seems scarcely possible to afford to the modern reader a more obvious example of the inconsistency of the methods formerly prevailing.

" person so advancing or paying the same, or during any other life to
" be nominated." (4 W. & M. cap. 3, s. 22, *vide* also 5 & 6 W. & M. cap.
" 5, s. 2). One hundred pounds thus securing four-
teen pounds annually, was virtually estimating the
value of an annuity on a single life at seven years'
purchase;—the first value assigned in the preceding
Table.

The most unpractised reader will at once per-
ceive, that until distinctions of age were recognized
as influencing the relative Annuity-values, all Life-
estimates were too imaginary and inconsistent to
be even worthy of the name of calculated results.

In the ensuing history it will be attempted to illus-
trate the various treatises that appeared in develope-
ment of such distinctions, and to thus indirectly
denote the authors to whom we are indebted for
the gradual establishment of the true doctrine of
Life-contingencies now forming so important an ele-
ment in the financial happiness of our own country.
In prosecuting this inquiry, it has been thought
preferable to merely proceed from date to date, in
lieu of any theoretical means of connection, and
to occasionally include such examples, both in ad-
vancement, and retrogression of general opinion,
that this, like the history of any other branch of
human knowledge, would lead us to expect, and
which indeed the particular history itself really ex-
emplifies and affords.

[1693] The utility of the distinction of ages in
matters of Life-measurement, though commonly dis-
regarded during the latter part of the seventeenth
century, was not, however, so wholly overlooked by

the more reflecting portion of the community, as not to prompt some endeavours to procure data in which the ages at death had been set forth. Apparently in this spirit Mr. Justell, a member of the Royal Society, obtained from Dr. Neumann some Bills of Mortality for the City of Breslaw, in which the distinctions of age had been registered for the years 1687, 1688, 1689, 1690, and 1691.

These Bills were carefully considered by the celebrated mathematician, Mr. (afterwards Dr.) Halley, who published the result of his investigation in Number 196 (1693) of the Philosophical Transactions, under the following title—" An Estimate of the Degrees of " Mortality of Mankind, drawn from curious Tables, of the Birth and " Funerals at the City of Breslaw, with an attempt to ascertain the " Price of Annuities upon Lives, by E. Halley, R.S.S."

Dr. Halley commences his paper as follows— " The Contemplation of the Mortality of Mankind, has besides the " *Moral*, its *Physical* and *Political* Uses, both which have been some " Years since, most judiciously consider'd by the curious Sir William " Petty, in his Natural and Political Observations on the Bills of " Mortality of London, own'd by Captain John Graunt. And since " in a like Treatise on the Bills of Mortality of Dublin. But the " Deduction from those Bills of Mortality seemed even to their Au- " thors to be defective : First, In that the *Number* of the People was " wanting. Secondly, That the *Ages* of the People dying was not to " be had. And Lastly, That both London and Dublin, by reason of " the great and casual Accession of Strangers who die therein, (as " appeareth in both, by the great Excess of the Funerals above the " Births) rendered them incapable of being Standards for this pur- " pose; which requires, if it were possible, that the People we treat " of, should not at all be changed, but die where they were born, " without any adventitious Increase from abroad, or Decay by Mi- " gration elsewhere.

" This Defect seems in a great measure to be satisfied by the late " curious Tables of the Bills of Mortality at the City of Breslaw, " lately communicated to this Honourable Society by Mr. Justell,

" wherein both the Ages and Sexes of all that die, are monthly de-
" livered, and compared with the Number of the Births, for five
" Years last past, viz. 1687, 88, 89, 90, 91, seeming to be done
" with all the Exactness and Sincerity possible.

" The City of Breslaw is the Capital City of the Province of Sile-
" sia,   *   *   *   and very nigh the Latitude of London.   It is
" very far from the Sea, and as much a Mediterranean Place as can
" be desired, whence the Confluence of Strangers is but small, and
" the Manufacture of Linnen employs chiefly the poor People of the
" Place, as well as of the Country round about.   *   *   *   For
" these Reasons, the People of this City seem most proper for a
" Standard; and the rather, for that the Births do a small matter
" exceed the Funerals.   The only thing wanting, is the Number of
" the whole People, which in some measure I have endeavour'd to
" supply, by the comparison of the Mortality of the People of all
" Ages, which I shall from the said Bills trace out with all the
" Accuracy possible." (Dr. Halley, Philos. Trans. 1692-93, vol. XVII.
p. 596.) *

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* Before proceeding further in the notice of Dr. Halley's commu-
nication, it may be well to premise, that in this as in future instances
the reader will necessarily have to refer for the precise nature of the
several details to the original publications.   The present writer
is too strenuous an advocate, for the Doctrine of Life-contingencies
being solely studied in connection with demonstrable calculations,
to recklessly present, even to a solitary reader, the chance of the
dangerous impression occurring, that such matters are capable of
being properly studied by merely cursory perusal.   The utmost
that can be safely offered in an historical Essay, is an attempt to
delineate the general principles on which different authors have suc-
cessively proceeded; with the occasional liberty, in a note, of intro-
ducing a casual formula, for the specific substantiation of some tex-
tual example, or when the history of the subject appears to reveal,
the origin of some calculative process not hitherto noticed by profes-
sional writers.

Dr. Halley, after premising some general remarks, eventually introduces a table of the following form:

*Age Curt.	Persons.	Age Curt.	Persons.	Age Curt.	Persons.	Age Curt.	Persons.	Age Curt.	Persons.	Age Curt.	Persons.	Age.	Persons
1	1000	8	680	15	628	22	586	29	539	36	481	7	5547
2	855	9	670	16	622	23	579	30	531	37	472	14	4584
3	798	10	661	17	616	24	573	31	523	38	463	21	4270
4	760	11	653	18	610	25	567	32	515	39	454	28	3964
5	732	12	646	19	604	26	560	33	507	40	445	35	3604
6	710	13	640	20	598	27	553	34	499	41	436	42	3178
7	692	14	634	21	592	28	546	35	490	42	427	49	2709
												56	2194
Age Curt.	Persons.	Age Curt.	Persons.	Age Curt.	Persons.	Age Curt.	Persons.	Age Curt.	Persons.	Age Curt.	Persons.	63	1694
43	417	50	346	57	272	64	202	71	131	78	58	70	1204
44	407	51	335	58	262	65	192	72	120	79	49	77	692
45	397	52	324	59	252	66	182	73	109	80	41	84	253
46	387	53	313	60	242	67	172	74	98	81	34	100	107
47	377	54	302	61	232	68	162	75	88	82	28		
48	367	55	292	62	222	69	152	75	78	83	23		34000
49	357	56	282	63	212	70	142	77	68	84	20	Sum Total,	

This table Dr. Halley wished, in effect, to be thus interpreted, namely, that of 1000 children, aged one year, as many would live to attain the older ages respectively as were denoted by the numbers set opposite to those ages.

From this it was to be argued that a child of one year old had precisely as many chances out of a thousand of attaining any particular age, as there were survivors represented by the table to attain such age.

Nor was this relation confined to children of one year old. For as the table was strictly a table representing consecutive survivors, so the prospection of a person of any specified age, attaining an older spe-

* The heading " Age Curt." appears from a preceding sentence in Dr. Halley's paper, to have been intended to signify " Age current."

cified age, was distinctly denoted by as many chances out of the number, commencing from such first age as there were survivors attaining the older age.

Two examples, by reference to the table, will suffice: thus, the chances of a child, aged one year, attaining thirty years of age, would be represented by 531 chances out of 1000; and the chances of a person aged twenty, attaining forty, would be 445 chances out of 598. In the same manner, *mutatis mutandis*, the chances of living and dying for all ages were easily ascertainable, the number of expectant survivors representing the chances of living, and the remainder, or the number who were not expected to survive, the chances of death.

These principles, self evident as they may appear, yet really only became so when the Table of Mortality was created and arranged; and thus Dr. Halley, by this simple arrangement, effected the resolution of many questions by comparative inspection, which previously were wholly incapable of even approximate answers.

The precedent table seems not only interesting to the general reader as the first complete " Table of Mortality" upon record, but also as constituting the first real step in the *art* of Life-measurement. It may further be added, as a fair tribute to Dr. Halley's skill, that 150 years of subsequent consideration by the most eminent mathematicians of Europe, have only tended to confirm the eligibility of the form primarily chosen, even the most modern tables of mortality being still arranged upon the same principle.*

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* It is left to the reader to determine what share of this tribute

Dr. Halley having thus defined the " chances of life," proceeds to illustrate the application of the table to the determination of the value of life annuities, and apparently selected 6 per cent. interest, as the rate most current in his own day.

He first indirectly advances that the worth of an annuity not dependent upon life, would be represented by adding together the several discounted values of the successive payments. But in a life annuity, as each payment would only be claimable upon the person's living to the respective dates, so each payment, would prospectively be worth such a proportion only of the discounted value as there was a proportionate chance of the person's living to the date in question. Now the chances of a person of any particular age living to the respective dates of payment were easily determinable, as already mentioned, by considering the relative number of yearly survivors, indicated by the table of mortality.

The value of the life annuity was therefore dependent;—firstly, upon deriving from interest tables or calculations, the discounted value of each yearly payment; and secondly, upon taking such a proportion only of each discounted value as there might be a proportionate chance of the payment being respectively claimed. Such separately reduced values being added together, would then constitute the total value of the life annuity.

This method of valuing life annuities, by ascertain-

should be referred to Mr. Graunt, in respect of his arrangement noticed in p. 15.

ing the value of each future payment separately, is obviously as demonstrably true in the present day, as in Dr. Halley's own time; but inasmuch as the labour of such subdivision for every age would be necessarily very great, the reader will not be surprised to learn that less elaborate methods have since been discovered. Dr. Halley, himself, remarked upon the operose requirements of his formula, but very frankly owned, (vide Phil. Trans. No. 198, Vol. 17, p.654,) that after several trials, he had not succeeded in devising any less laborious method. He had, however, the perseverance to calculate the results for every fifth age as embodied in the accompanying table, which, from its constituting the first Annuity-table for different ages, seems no less interesting as the primary step in the art of Life-finance, than the preceding table was, in that of Life-measurement.

TABLE.

Age.	Years Purchase.	Age.	Years Purchase.	Age.	Years Purchase.
1	10,28	25	12,27	50	9,21
5	13,40	30	11,72	55	8,51
10	13,44	35	11,12	60	7,60
15	13,33	40	10,57	65	6,54
20	12,78	45	9,91	70	5,32

Dr. Halley, in connection with this Table, observes, " This shews the great Advantage of putting Money into the present " Fund lately granted to Their Majesties, giving 14 per Cent. per " Annum, or at the Rate of 7 Years purchase for a Life; when young " Lives, at the usual Rate of Interest, are worth above 13 Years pur- " chase. It shews likewise the Advantage of young Lives over those " in Years; a Life of Ten Years being almost worth 13½ Years Pur- " chase, whereas one of 35 is worth but 11.''*

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* Dr. Halley, deeming it necessary to remark that Annuities on

Dr. Halley then considers the chances appertaining to two or more lives, and shows that the same principle of investigation already applied to single lives, should be extended to the more compound cases; for the seperate yearly values being ascertained, and added together, would similarly constitute the total value of such Annuities, care, however, being taken that each yearly value was previously compounded of the chances of the different lives under consideration, according to the terms of the particular case to be solved. Some geometrical illustrations are then added; but as his previous elucidations really needed no such extraneous aid, being already sufficiently explicit, it is unnecessary to delineate such figures in the present instance.

It may be well, as a conclusion, to enumerate the uses which Dr. Halley thought his Table was capable of affording in his own day.——" Thus it appears " that the whole people of Breslaw does consist of 34,000 souls, being " the sum total of the persons of all ages in the table. The first Use " hereof is to shew the proportion of men able to bear arms in any " multitude, which are those between 18 and 56 years. * *

" The Second Use of this table is to show the differing degrees of " mortality, or rather vitality, in all ages. * *

" Use III. If it be inquired at what number of years, it is an even " lay that a person of any age shall die, this table readily performs it : " for if the number of persons living of the age proposed be halfed, " it will be found by the table at what year the said number is re- " duced to half by mortality; and that is the age to which it is an " even wager that a person of the age proposed shall arrive before he " die; as, for instance:—A person of 30 years of age is proposed, the " number of that age is 531, (vide table, p. 25) the half thereof is " 265, which number I find to be between 57 and 58 years, so that

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" Young Lives " were more valuable than upon those " in Years," sufficiently pourtrays how little attention must have been previously devoted to the subject.

" a man of 30 years may reasonably expect to live between 27 and 28
" years.

" Use IV. By what has been said, the price of insurance upon
" lives ought to be regulated; and the difference is discovered between
" the price of insuring the life of a man of 20 and 50. For example:—
" It being 100 to 1, that a man of 20 dies not in a year, and but 38
" to.1, for a man of 50 years of age.*

" Use V. On this depends the valuation of annuities upon lives.
" *   *

" Use VI. Two lives are likewise valuable by the same rule * *
" Use VII. Three Lives are proposed *   *   *   *

Dr. Halley, in a supplemental number, adds a re-
flection, which, though not statistical in itself, is cer-
tainly not among the least results to be wished for
from such inquiries—" Besides the uses mentioned, it may,

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* The reader may be surprised at this early mention of " Insurance
for Lives," but it should be remembered that the only species of Life
Assurance current in Dr. Halley's time was of a similar nature to Fire
and Ship Insurance, namely, for a particular term; thus, in a cotem-
porary law report (Salkeld, vol. 2, fol. p. 625) is the following, " Sir Ro-
" bert Howard's Case, Trin. 11 Will. 3, B. R. at the sittings at Guild-
" hall. A Policy of Assurance was made to ensure the life of Sir Ro-
" bert Howard for one year from the day of the date thereof, the policy
" was dated on the 3d day of September 1697, Sir Robert died on the
" 3d day of September 1698, about one o'clock in the morning. Et.
" per Holt C. J. In an action thereupon it was ruled at the sittings at
" Guildhall—First, That from the day of the date excludes the day,
" but from the date includes it, so that the day of the date is excluded.
" Secondly, That the law makes no fraction in a day, yet in this, he
" dying after the commencement and before the end of the last day, the
" Insurer is liable because the Insurance is for a year, and the year is
" not complete till the day be over." A distinction equally applicable
" to Fire and Ship Insurance.

Dr. Halley therefore, obviously, merely referred to the fallacy of the
current system as applied to Lives, and that different ages really causing
different hazards, the year's insurances ought to be respectively charged
in the same ratios as the chances of yearly deaths.

" perhaps not be an unacceptable thing to infer from the same tables
" how unjustly we repine at the shortness of our lives, and think our-
" selves wronged if we attain not old age; whereas, it appears hereby
" that the one half of those that are born are dead in seventeen years
" time, 1238 being in that time reduced to 616; so that, instead of
" murmuring at what we call an untimely death, we ought, with
" patience and unconcern, to submit to that dissolution which is the
" necessary condition of our perishable materials, and of our nice
" and frail structure and composition, and to account it as a blessing
" that we have survived, perhaps by many years, that period of life
" whereat the one half of the whole race of mankind does not arrive."
(Dr. Halley Phil. Tr. No. 198, Vol. 17, p. 654, et seq.)

With the above quotation we close this passing notice of Dr. Halley's celebrated " Attempt to ascertain the price of Annuities upon Lives," probably the most important contribution to the since well-established Doctrine of Life-contingencies.

[1694.] The succeeding example of the valuation of life is more typical of a century before Dr. Halley's " Estimate," than the year following, as will be evidenced by stating, that in 1694 a money act was passed, in which Life-Annuity values were thus declared—" One life only * * a yearly annuity rent, or pay-
" ment of fourteen pounds of lawful English money for every hundred
" pounds. *   * two lives, an yearly annuity * * of twelve
" pounds for every one hundred pounds; and * * for three
" lives, a yearly annuity of ten pounds * * for every hundred
" pound." (5 W. & M. cap. 20, s. 34.) Consequently, a single Life-Annuity was merely estimated at 7, two Lives at 8½, and three Lives at 10 years purchase. (vide also 6 & 7 W. 3, cap. 5, s. 5, and 7 & 8 W. 3, cap. 2, s. 4).

[1695.] In the following year (1695) an act was passed, to levy duties upon Marriages, Births, and Funerals, &c. the duties varying with the rank of the parties.

The act specifies fifty-six distinctions in such mat-

ters; the Duty for every burial was 4s. with an additional rate, of which the greatest was £50, being that for the death of a duke or duchess. The Duty upon each birth was 2s. with an additional rate commencing from £30. The Duty upon every marriage was 2s. 6d. with an additional rate commencing from £50.

The means provided to ensure the fulfilment of the returns still constitute one of the most stringent measures for registration on our statute books, and, were it possible to overlook the venality of the real object, might lead to the supposition, that statistical inquirers were vigorously assisted by the Legislature, even as early as 1695. The statute, after detailing the duties, &c. enacts——" That all persons in holy orders, * * " do, within their respective parishes, keep a register, in writing, of " all * * persons, married, buried, christened, or born, in his " or their respective parishes, * * and if any such parson, or " minister, shall refuse or neglect to keep a true register therof, as " before is directed, such parson, or other minister, so offending, shall " forfeit the sum of one hundred pounds." (6 & 7 W. 3, cap. 6, s. 24.)

[1696.] As the reader may feel some curiosity to learn the numerical results of such an enactment, the comparative details are here inserted, the means of supplying them being fortunately accessible; Mr. Gregory King, Lancashire Herald, having in 1696, composed a manuscript, in which most of the statistical results occurring during his own time were diligently recorded.

The following is Mr. King's account as it occurs in his " Natural and Political Observations," first reprinted by Mr. Chalmers from the manuscript in the British Museum, and inserted as an Appendix to his " Estimate."

ON THE PRESENT DUTY ON MARRIAGES, BIRTHS, AND BURIALS,

ACCOUNTING THE PEOPLE TO BE 5,400,000 SOULS.

AT A MEDIUM IN TIME OF PEACE.

Yearly		In all	Com. Duty		Anno. 1695, Thus :	£
Burials	1 in 32	170,000 at 4s. each	34,000	1 in 29½	183,000	36,600
Births	1 in 28	190,000 at 2s. "	19,000	1 in 30½	177,000	17,700
Marriages	1 in 132	41,000 at 2s. 6d. "	5,125	1 in 140	39,000	4,875
Batchelors	1 in 40	140,000 at 1s. "	7,000	1 in 40	140,000	7,000
Widowers	1 in 200	27,000 at 1s. "	1,350	1 in 200	27,000	1,350
		In all	£66,475			67,525

OMISSIONS, FRAUDS, AND INSOLVENTS.

In Burials	-	6 per cent	—	10,000 at 4s.	each	£2,000
Births	-	3 "	—	6,000 at 2s.	"	600
Marriages	-	2½ "	—	1,000 at 2s. 6d.	"	125
Batchelors	-	10 "	—	14,000 at 1s.	"	700
Widowers	-	5 "	—	1,500 at 1s.	"	75
				In all		£3,500

EXCUSED BY RECEIVING ALMS.

In Burials	- - - - - -	00,000 - - - - -	£00,000
Births	- 30 per cent. —	60,000 at 2s. each	6,000
Marriages	- 10 " —	4,000 at 2s. 6d. "	500
Batchelors	- 5 " —	7,000 at 1s. "	350
Widowers	- 20 " —	5,000 at 1s. "	850
		In all	£7,100

So the common Duty comes to - - - - - - - - £56,475

And the Deductions - - - - - - - - - - - 10,600

Whereby the net produce of the common Duty is - - - - £55,875

The Persons charged for Quality are about 1 in 10 of the whole; viz.

Burials	- - - 17,000	—	at 14s. each	£11,900
Births	- - - 19,000	—	at 8s. "	7,600
Marriages	- - 4,000	—	at 10s. "	2,000
Batchelors	- - 14,000	—	at 5s. "	3,500
Widowers	- - 3,000	—	at 5s. "	1,500
	In all, for Quality			£26,500

Omissions, Frauds, and Insolvents, in Quality,
a 20th part, or - - - - - - - - } 1,325

Whereby the neat produce for Quality is- - - - 25,175

And the neat Produce of the Common Duty - - 55,875

So the neat Produce, in all, should be - - - - - 81,050

Whereas it is given for - - - - - - - - - - £130,000

The manuscript above alluded to contained several other curious tabular statements, from which we select the following, as being probably the first table in which the ages of the people collectively were attempted to be defined.

IV. The several Ages of the People.

That the Yearly Births of the Kingdom being 190,000 Souls:

Those under 1 year old	-	-	are	170,000	90,000	80,000
Those under 5 years old	-	-	are	820,000	415,000	405,000
Those under 10 years old	-	-	are	1,520,000	794,000	756,000
Those under 16 years old	-	-	are	2,240,000	1,122,000	1,118,000
Those above 16 years old	-	-	are	3,260,000	1,578,000	1,682,000
Those above 21 years old	-	-	are	2,700,000	1,300,000	1,400,000
Those above 25 years old	-	-	are	2,400,000	1,150,000	1,250,000
Those above 60 years old	-	-	are	600,000	270,000	330,000

So that the number of communicants is, in all - - - - 3,260,000 souls.

And the number of fighting men, between 16 and 60, is - - 1, 310,000

That the batchelors - - - - - are about 28 per cent. of the whole.
   Whereof those under 25 years - - - - - - are 25$\frac{1}{2}$ per cent.
   And those above 25 years - - - - - - - - are 2$\frac{1}{2}$ per cent.

That the maidens - - - - - - are about 28$\frac{1}{2}$ per cent of the whole.
   Whereof those under 25 years - - - - - - are 26$\frac{1}{2}$ per cent.
   And those above 25 years - - - - - - - - are 2 per cent.

That the males and females, in the kingdom in general, are aged, one with another, 27$\frac{1}{2}$ years.

That in the kingdom in general, there is near as many people living under 20 years of age, as there is above 20. Whereof, one half of the males is under 19 years, and one half of the females is under 21 years.

AT A MEDIUM.

That the Husbands are aged 43 Years a piece, which, at 17$\frac{1}{4}$ per Cent. makes 742					
The Wives	"	40 Years a piece,	"	17$\frac{1}{4}$	"	690
The Widowers	"	56 Years a piece,	"	1$\frac{1}{4}$	"	84
The Widows	"	60 Years a piece,	"	4$\frac{1}{2}$	"	270
The Children	"	12 Years a piece,	"	45	"	540
The Servants	"	27 Years a piece,	"	10$\frac{1}{2}$	"	284
The Sojourners	"	35 Years a piece,	"	4	"	140
At a Medium,	27$\frac{1}{2}$			100 Persons	2,750

(Gregory King, Natural and Political Observations, 1696. § IV.)

Mr. Gregory King appears to have chiefly founded his conclusions (§ IV.) upon the returns afforded by the books, in connection with the various enactments for raising money by Poll-taxes, on different occasions during his own time. (*vide* 1 W. & M. sess. 1, c. 13, ad. 4 & 5 W. & M. c. 14.) His work abounds with old and curious statistical details, and is, in every respect, well worthy of the reader's consultation. The two tables, however, above inserted, seem, to comprise all that it is necessary to offer on the present occasion.

[1703] In 1703, a money-act was passed, in which life annuities were granted upon a similar principle to that in the previous acts, but upon less favourable terms; in place of seven, eight and a half, and ten years' purchase, the prices being, " for one life, at the rate of nine years' pur-" chase; or, for two lives, at the rate of eleven years' purchase; or, " for three lives, at the rate of twelve years' purchase." (2 & 3 Anne, " cap. 3, s. 10 ; *vide* also 3 & 4 Anne, cap. 2, s. 3.)

This persistance in imaginary estimates, notwithstanding Dr. Halley had, ten years previously, illustrated the importance of distinction as to age, &c., pourtrays that the new Doctrine of Life-valuation was as yet but little appreciated, being probably overlooked, or deemed still more fanciful than even the ordinary methods then in use.

[1705-6] In confirmation of the general disregard as to the ages of annuitants, &c. an association, in which such distinctions were merged, was formed by private agreement, (Jan. 24th, 1705,) under the title of " The Amicable Society for a perpetual Assurance Office." In the following year, (July 25th,

1706,) the Society obtained a charter from Queen Anne, and were thereby duly incorporated.

We learn from this charter, that the Society were not to exceed 2000 in number: that each member was to pay, ten shillings entrance money, and ten shillings monthly, or an equivalent sum quarterly, with one shilling quarterly, in either case additional.

The annual subscription was thus six pounds four shillings, of which six pounds appear to have been intended for contingencies, and four shillings, in addition to the ten shillings entrance money, for expenses. In respect of these payments, which were equally chargeable to members of all ages, each contributor was to receive " one Policy of Assurance, under the " seal of the said Corporation, entitling each respective nominee or " nominees, to such dividend or dividends, and in such manner as " hereinafter mentioned." (Charter, Clause XV.)

Such dividends were apparently regulated, by one-sixth part of the contributions being annually distributable, in equal proportions, among the nominees of such policies as might constitute claims in respect of deaths during each year respectively.* Thus, in the first year, two thousand pounds were to be distributed, being one-sixth part of two thousand contributions of six pounds each; in the second, four thousand pounds, (being one-sixth of the two years' subscription); and so, successively and proportionately, until the sixth year, when the dividend was to be fixed by a general court, &c.; but that in case two thousand members should not be prima-

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* " One-sixth part " appears to have been allotted as representing £1 of the £6, of which each contribution chiefly consisted.

rily associated together, then a proportionate dividend was to be observed in respect of such lesser amount of contributions accordingly. (*vide* Clauses XXII.-III.)

From the above detail, it is obvious, that such regulations did not constitute a Life Assurance Association in the modern meaning of the term, but rather what may be called a Mortuary-tontine, in which an annual produce was to be divisible among the nominees of the deceased, in place of among the survivors, as in ordinary Tontine cases. The denomination, " perpetual Assurance Office," was probably therefore, simply employed in a legal sense, to signify that the destined two thousand original contracts, or nominee shares, in place of becoming finally cancelled by death, would like land or real estate, be perpetually " assured " or " conveyed " to various successors.

In the foregoing account of the Amicable Society it may be seen, by what gradual means the important system of pecuniary provision against the evils of premature death, was being established. Without observance of the distinction of age;—without any definite notion as to the value of each nominee's expectant share of the mortuary-dividend;— yet an assembly of private individuals, as early as 1705-6, considerately felt, that, if the general notions as to the tenure of life, admitted of being acted on to provide an annuity for an individual himself; so pecuniary matters might be equally arranged, to provide support, for such members of his family, as might eventually survive him.

To the founders of the Amicable Society, we thus owe the first emblem of that consideration for the welfare of kindred, happily so strongly confirmed as a general feeling by the numerous Life Assurance engagements that have been and still are daily entered into ;—

"Humberston Baron, John Hartley, William Spenccly, Richard "Musgrave, and others, have, by their humble petition, represented "unto us, that they have agreed upon and entered into a voluntary "society for the mutual benefit and interest of every person that shall, "at any time, be a member thereof, in order to provide for the Widows, "Children, and other relations, after a more safe, ecrtain, and advan-"tageous method, than any that hath hitherto been thought of; by an "Amicable contribution, according to·eertain articles or agreements "entered into by the said Petitioners, for the purposes aforesaid.

"And it has been humbly certified unto us, that their design will "be of singular use and relief to many Families, by providing for great "numbers of Widows and Orphans who might, probably, be other-"wise left wholly destitute of a maintenance by the sudden Death of "those on whom they depend

"And the said Petitioners have therefore humbly prayed us to In-"eorporate them, and all others that are or shall be concerned, in "the Soeiety of Perpetual Assuranee Office, that they may be enabled "to purehase Lands, to be settled as a security for payment of the "elaims and other uses mentioned in the aforesaid artieles.

"Now know ye that we (are) graciously pleased to gratify the said "Petitioners in their said request, and to encourage the said under-taking." (Charter granted by Queen Anne, July 25th 1706, c. 1-3.)*

[1710] In 1710, a little treatise was published entitled "*Clavis Usuræ*; or a Key to Interest, &c., to which is "added, Rules to be observed in estimating the value of Annui-"ties or leases and Insurance for Lives, &e., by J. Ward."

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* The casual reader will probably be pleased to learn that "*the* "*Amicable Society for a Perpetual Assurance Office*," is still existent and flourishing, and by the numerous facilities and improvements in-troduced from time to time, constitutes one of the most eminent Life Assurance Soeieties of the present day.

The only portion of Mr. Ward's work that is devoted to " Annuities and Insurances for Lives," is nearly wholly composed of extracts from Sir. W. Petty's Use of Duplicate Proportion, and Dr. Halley's Paper, each already referred to, (*vide* pp. 17, 23.) As, however, the mere fact of the distinction of ages being admitted in both the papers selected, pourtrays that the subject of Life-measurement was beginning to be more or less appreciated, it may be well to insert some quotations from Mr. Ward's little book, as illustrating the opinions of his own time:

" The Way generally used in Buying of Annuities, or Letting of
" Leases for Lives, is only by an imaginary Valuation, grounded upon
" Custom, and not upon any Consideration that is had to the Age of
" the Persons whose Lives are to be inserted in the Lease, &c. 'Tis
" true indeed, that there can be no certain Rules prescrib'd for their
" true Valuation, because the Lives of all Mankind are uncertain;
" and 'tis possible, and daily seen, that a Young Man may Die before
" one of a greater Age: But yet there is a greater Probability of a
" Young Man's Living longer than an Old one; And not only so, but
" there's a Proportional Likelihood of the Length of Mens Lives,
" according to their different Degrees of Age; The which being duly
" considered, must needs be found of good Use in Estimating the
" Value of Annuities or Leases for Lives, much better than by a meer
" Guessing only as usual; And that such a Proportional Likelihood
" is worth the Consideration, will appear from what follows." (p. 104)

He then recites Sir W. Petty's Use of Duplicate Proportion, afterwards adding by way of comment:

" Thus you have a Learned Gentleman's Opinion concerning the
" Likelihood of the Length of Men's Lives, according to the Rules of
" *Duplicate Proportion*; which was a very ingenious Thought of His;
" But I must beg Pardon, that I cannot agree with Him in that part of
" it, which asserts, That 21 Years is the Age for whose Life a Lease is
" the most valuable: For although 'tis true, that according to our
" Law, a Man is said to be then at his Perfect or Full Age, as to the
" Enjoyment of an Estate, or Managing his Affairs without a Guar-
" dian, &c. Yet I should rather adhere to the Close of His Discourse,

" wherein He says, That He found, that if the Sum of all the Ages
" of the 330 Souls (in a certain Parish) being Divided by their Num-
" bers, made the Quotient between 15 and 16. Whence I take 16 to
" be the Age, for whose Life a Lease is the most valuable; And upon
" that Supposition I have Calculated the following Table, according
" to the aforesaid Rules of *Duplicate Proportion*.

Ages.	Year's Purch.	Ages.	Year's Purch.	Ages.	Year's Purch.
1	2,50	26	7,20	51	5,04
6	5,51	31	6,46	56	4,84
11	7,46	36	6,00	61	4,60
16	9,00	41	5,62	66	4,43
21	7,85	46	5,13	71	4,27

" This Table shews, by Inspection, the Value of every Five Years
" of any single Life, from the Birth to 71 Years old. Supposing that
" any Annuity, or Lease, &c. is really Worth Nine Year's Purchase
" for One Life; which is according to the Rate that the Annuities
" settled by a late Act of Parliament for Lives, were valued at, and
" from thence the rest are computed. (p. 106)

He then notices Dr. Halley's " Estimate," and after inserting the relative Table of Annuities, adds:

" This *Table* being of the same Nature with that in *Page* 107, (*i. e.*
" the Table above), there needs no other Explanation or Example to
" shew its Use, than what has been already said about that *Table:*
" Only here I must again beg leave to give my Opinion about the
" *Difference* of the *Proportions* in the Two *Tables*; which is, that as
" the *Table* in *Page* 107, may not be thought a sufficient Guide to be
" depended upon in Estimating the Length of Men's Lives, &c.
" because its only deduced from the bare Rules of Art, *viz.* that of
" *Duplicate Proportion*; so on the other Hand, I doubt the Estimates
" of the Value of any Annuity taken from this *Table*, (*i.e.*Dr. Halley's)
" will be found too great in this Country, (viz. in England) which I
" much fear hath not so Good or *Salubrious* an *Air*, as that at the
" City of *Breslaw*, from whence these *Calculations* are drawn: But
" if in Imitation hereof the Curious in other *Cities* and *Large Towns*
" would attempt something of the same Nature; Then without all
" doubt this Method of Estimating the Probability of the *Length* of
" Men's Lives, would prove the Best, and become more universally
" Useful than can be expected from this one single Instance, more

" especially if such Observations were continued for any considerable
" Time, as 20, or 30 Years successively." (Ward's Clavis. Usuræ,
1710. p. 111.)

The above quotations from a little Interest-book
published in 1710, sufficiently illustrate that how-
ever much the subject of distinctive Life-valua-
tion might be generally overlooked, yet that it was
gradually attracting the attention of occasional
writers.

The Life Annuity Acts previously mentioned, and
the establishment of the Amicable Society, appear to
have originated several speculative attempts to
establish lotteries in connection with the chance of
Life, an Act being passed, in which was declared
" That every person or persons who after the Eighth day of March,
" 1710, shall erect or set up any Office or place for making Assurances
" on Marriages, Births, Christenings, and Service, or on any of them,
" shall forfeit for every such offence the sum of Five Hundred Pounds."
(9 Anne, cap. 6, s. 57.)

These, and similar attempts to establish Lotteries
in relation to Life-Contingencies, were more or less
to be expected, if it be considered that the only
Society then existing, that guaranteed reversionary
sums was indirectly founded upon the principle of a
Mortuary-tontine. For even supposing that the
Amicable Society had not been restricted in number
to two thousand members, and thus might have com-
prised a larger circle, yet it was evidently not the in-
tention of its founders, to include money contracts
specifying *particular* sums, to be payable at death, but
merely to divide the yearly income among the nomi-
nees of recently deceased members.

The simple guarantee, therefore, of a certain sum
to be payable at death, in respect of a certain con-

tribution to be received during Life, had yet to be undertaken as the province of a definitive Establishment, and thus Lottery Offices, and personal Life-wagers, probably supplied accommodation to this class of cases.

[1720-21] In 1720-21, two Marine Insurance Offices (the Royal Exchange, and the London) applied for Charters, to enable them to comprise Fire and Life Insurance, covenanting to supply respectively the remainder-sums of £180,750 in fulfilment of the £300,000 guaranteed to the Crown, in respect of the Shipping Charters of the previous year, (6 Geo. I, cap. 18); and in relation to such new guarantee were granted extra Charters accordingly;—£150,000, however, of each subsidy being ultimately abandoned, (7 Geo. 1, cap. 26, sec. 26).

The purport of these Companies appears to have been to merely undertake Life Assurance contracts among other classes of popular risks, and if the current notions then existing, as to the uncertainty of Life, be considered, it will become evident that it was only by Companies having a Capital at command that *specific* reversionary sums at so early a date could be warrantably guaranteed.*

The reader will thus see that two species of mor-

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* The Union Office was established in 1714, but being unincorporated, the precise date of the Company's undertaking Life, in addition to Fire Insurance, does not become apparent.

It is scarcely necessary to add, that the Union and other highly-respectable Companies above mentioned have long since abandoned the old methods of valuation, and are no less distinguished for their attention to statistical data than the most eminent of their subsequent cotemporaries.

tuary contracts had become accessible to the public;
one in which the reversion fluctuated in amount
from year to year; as presented by the Amicable
Society; the other, in which a definite sum was named
in each policy, as offered by the " Insurance" Com-
panies.

In both cases, however, distinctions of age were
overlooked, and continued to be so, until authors had
so fully substantiated the trustworthiness of the
doctrine of Life-contingencies, as to cause the esta-
blishment of societies for Annuity and Assurance
purposes, in which distinctions of age, among other
statistical elements, were duly recognized and appre-
ciated.

The relative works that appeared prior to the date
of establishment of the first Life Assurance Society,
strictly so called, will therefore now form the en-
suing topics of illustration; it being, however, well
to premise, that as Life Assurance formulæ were
eventually derived from annuity calculations, so it
was not until an after-period, that Life Assurance
attracted notice as a distinct subject.

[1725]   The first treatise appeared in 1725, un-
der the following title: " Annuities upon Lives: or the Valua-
" tion of Annuities upon any number of Lives, as also of Reversions.
" To which is added, an Appendix concerning the Expectation of Life,
" and Probabilities of Survivorship;" by Abraham De Moivre, F.R.S.

De Moivre commences his Treatise by observing,
" The method of Calculating the Values of Annuities upon Lives,
" seems never to have been perfectly understood 'till Dr. Halley first
" published the Rules of it; which may be seen in the Philosophical
" Transactions, No. 196.

" Although there is some ground to believe that what he writ upon
" that Subject was partly occasional; yet there appears in it the

" same exquisite Judgment and Sagacity that has always shewn it-
" self in all his other Productions.

" As this Calculation chiefly depends on the various Degrees of
" Probability which Lives of different Ages have to continue in Being,
" so it was previously necessary that a Table should be constructed
" which might at one View exhibit the State of Life in Respect to its
" probable Duration: Such a Table was according constructed in the
" best Form that could be contrived, out of five Years Observations
" upon the Bills of Mortality of Breslaw the Capital of Silesia.

" But notwithstanding the Pains taken by that excellent Mathe-
" matician in fitting a true Algebraic Calculation to his Table of
" Observations, yet he was sensible that both his Table and Calcula-
" tions, were capable of further Improvement; of this he expressed
" his sense in the following Words: Were this Calculus founded on
" the Experience of a very great Number of Years, it would very
" well be worth the while to think of Methods to facilitate the Com-
" putation of Two, Three or more Lives.

" From whence it appears that the Table of Observations, being
" only the Result of a few Years Experience, is not so entirely to be
" depended upon as to make it the Foundation of a fixt and unalter-
" able Valuation of Annuities for Life, and that even admitting such
" a Table could be obtained as might be grounded on the Experience
" of a very great Number of Years, still the Method of applying it
" to the Valuation of several Lives would be extreamly laborious,
" considering the vast Number of Operations that would be requisite
" to combine every year of each Life with every Year of all the
" other Lives.

" I own, that in the Method which I have taken to make the Cal-
" culation of Lives easy, I have not made use of any other Table
" than of that which has been constructed by Dr. Halley; for which
" Reason, I am contented it should be thought that the Values of
" Annuities, as deduced from my Calculation, are still liable to some
" Exceptions, and need not therefore prejudice any established Prac-
" tice of Estimation.

" Wherefore all I can in Justice pretend to, is to have contrived
" such Rules as will make the Valuation of Lives easy, whenever it
" shall so happen that a Table is produced that can be depended
" upon, as to the Sufficiency of the Number of Observations on
" which it is built, the Accuracy of the Observers, and the Skill of
" those who construct the Table from the Observations."

He primarily details, Dr. Halley's method of pro-
ceeding, and fully corroborates its correctness, but
adds, that it was not, however, really necessary to
calculate the separate value of each annual payment
for all ages:* for when the value of a Life Annuity
had been determined for one age; the value for the
age one year less might be incidentally deduced.

The reasoning by which this important theorem
was substantiated, may be cursorily illustrated, by
supposing the annuity in question to be £1 per an-
num, and the value of a similar annuity, at some par-
ticular age, to have been already ascertained by Dr.
Halley's process.   De Moivre, then in effect, ar-
gued, that though the value of the Annuity at the
age one year less, would be certainly more than at
the subsequent age, yet that the additional value
could not amount to £1 ; because even supposing the
person certain to attain the subsequent age, yet that
the £1 to be then due would have to be discounted in
computing its *prospective* value.

Moreover, as the worth of the subsequent annuity,
was also, as it were, one year distant, so the ordi-
nary value of such subsequent Annuity would also
have to be discounted in respect of the intervening
year.   But, in lieu of its being certain, there was
only a probable chance of the person's attaining
the extra year of age, consequently, the extra £1
and future annuity would be exactly worth such a
proportion only of their discounted values as there
was a proportionate chance of the person's sur-
viving the one year in question, and thus the value

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* *Vide* notice of Dr. Halley's paper in p. 27.

of the annuity at the lesser age in question, was indirectly ascertained. By this simple method of proceeding, De Moivre contrived to determine the value of the required annuity, by merely considering the intermediate chance of living from the one age to the other, whereas Dr. Halley had previously supposed it to be necessary to recommence for every age, and to calculate the value of each payment in succession.

This excellent and original suggestion by De Moivre is not only trustworthy in itself, but has afforded the fundamental formula by which nearly all our well-known tables of Life Annuities, including the Northampton and Carlisle, have been verified or computed; the procedure being, to commence at the greatest age in the Table of Mortality, and to retrogressively calculate the Annuity values from age to age, by yearly intervals in the manner referred to. *

De Moivre having thus pourtrayed the method of

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* As the origination of this formula has hitherto been generally attributed to the celebrated mathematician, Thomas Simpson, whose Treatise on Annuities was not published till seventeen years later, (1742), it may be necessary to state the plea upon which the idea is here claimed for De Moivre. This will be best effected by quoting his own words, merely premising that in a preceding sentence he had determined, by Dr. Halley's method, that the value of £100 per annum during the life of a person aged 30, money yielding 5 ₽ cent. interest, was " about £1300."

" Although the Number of *Operations* that are to be made in order " to estimate one single Life be very numerous, yet that once being " once determined, the Value of the next younger Life may easily " be obtained.

" Thus suppose it were required to determine the Value of a Life

obtaining particular results, then proceeds to investigate the character of the natural law upon which such results might be presumed to depend. Speculating as to the character of such law, he says, " In order to it, we may consider that whatever be that Law which

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" of Nine and Twenty, from the given Value of a Life of Thirty, we " may argue thus,

" The present Value of the first Year's Rent, upon a Certainty of " its being received, would be 95.23; but its being received is only " probable according to that Degree of Probability which is measured " by the *Fraction* $\frac{531}{539}$, whereof the present value of that Rent upon " the Contingency of the Life's failing in the first Year, is the Pro- " duct of 95.23 by $\frac{531}{539}$, that is 93.82.

" Likewise, the present Values of all the succeeding Rents will be " £1300, when once the Age of Thirty is attained, as we have seen " before.

" But it being uncertain whether that Age will be attained to, that " Value ought to be diminished in the Proportion of 531 to 539, " which will reduce it to 1280.7, and this Sum the Purchaser might " engage should be paid after the Expiration of one Year whether he " lived or not, for the Expectation of enjoying the Annuity after the " Age of Thirty.

" But if the Sum of 1280.7, which is payable after one Year, be " discounted at the Rate of 5 *per Cent.* its present Value will be re- " duced to 1219.7, which being added with the present Value of the " first Year's Rent, viz. 93.82, the Sum will be 1313.52, which will " be the Value of an Annuity for a Life of Nine and Twenty.

" By help of the Method hitherto explained, those that will take " the Pains may, if they think fit, compose Tables of the several " Values of those Annuities for any Age proposed, and for any Rate " of Interest that shall be fixed upon." (De Moivre's Treatise on Annuities, 1725. p. 6, *vide* also pp. 24, 27.)

The reader conversant with such matters will at once detect, in the above quotation, the idea afterwards embodied by Simpson in the seventh Corollary to his first Problem, $K = \overline{1+P} \times \dfrac{Q \, R \, S \, \&c.}{r \, Q_1 \, R_1 \, S_1 \, \&c.}$ (*vide*

Simpson's Treatise on Annuities, 1742. p. 18, comparing also p. 8 with the above quotation from De Moivre.)

" is observed by Nature in the perpetual Decrements of human Life,
" that Law must, conformably to all the other Laws of Nature, be
" such as to proceed regularly, at least for some short Intervals of
" Time." (De Moivre, 1725. p. 8.)

In connection with this view, he then examines
Dr. Halley's Table year by year, and finding that
the differences between the numbers were tolerably
constant, adds, "Let us therefore consider, 1º what would be the
" result of an *Hypothesis* that makes the Probabilities of Life to de-
" crease in *Arithmetic Progression*: 2º how far the Calculations de-
" duced from it agree with the Tables : 3º what Corrections are ne-
" cessary to be made to it where it varies from the Tables."
(*ibid.* p. 11.)

For De Moivre's precise method of investigating
such propositions, and his subsequent consideration
of 27 different problems, the reader is necessarily
referred to the treatise itself, it being sufficient for
the present occasion to merely offer, in illustra-
tion, the general tenour and results of the theory in
question.

The theory thus proposed, and since celebrated,
as " De Moivre's Hypothesis," consisted of the as-
sumption, that any specified number of persons born
would be subsequently decreased, from age to age,
by some uniform number of deaths. From this it
was evident, that as the number of deaths was sup-
posed to be invariable, so such number would annu-
ally be in greater proportion to the diminishing
number of survivors, and thus consistently represent,
at successive ages, a yearly decrease in the proba-
bilities of life.

To render such an hypothesis capable of yielding
numerical results, it became necessary to select a
commencing rate of mortality. In order to appor-
tion this, without unduly affecting the successive

durations of Life, among the subsequent survivors from age to age, De Moivre sought to ascertain in how many years a number of persons cotemporaneously born might be presumed to become extinct.

He accordingly referred to Dr. Halley's table, and finding, that of the 1000 persons represented at one year of age (*vide* p. 25), 531 attained the exemplar age of 30, and that the 531 became reduced to about one-half (262) in 28 years, he thought it reasonable to suppose, that an extra period of 28 years would extinguish the remainder, and consequently the age of 86 (30+28+28) be fairly presumed, for calculative purposes, as the limit of life. (*ibid.* p. 10.)*

The boundary being thus fixed, the annual mortality for all ages, by aid of the hypothesis, was thereby indirectly specified, because, as the *number* of deaths was supposed to continue uniform, so one eighty-sixth part of the specified number of persons *born* would presumably have to die in each successive year, to finally fulfil the total extinction

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* " I have thought it convenient to fix the utmost Limit * * in " the 86th Year, whereby I have nearly conformed myself to the Table " of Observations; but as some exeeptionable Consequences might " perhaps flow from too rigid an Applieation of that Principle to those " Years of Life that border upon its Extremity; it will not be im- " proper to declare, that my Meaning, in fixing that Term, imports " no more than this, viz. that the Time of contraeting for Annuities " being commonly very remote from the 86th Year of the Life pur- " chased, it is not likely that any Consideration will then be given " for the Chanee of reeciving the Rent of that Year, which will pro- " duce the very same Conclusions in Theory, as if the Extremity of " that Year were never attainable." (*ibid.* p. viii.)

previous to attaining the age of 86, the declared " limit" of life.*

De Moivre's Hypothesis, in its practical form, as connected with the Breslaw Table, was therefore thus expressible, viz. of every eighty-six persons born one was supposed to die annually, from age to age, till all were extinct.†

It may be well to offer the relative Annuity-values as determined by the correct but laborious method of Dr. Halley, (*vide* p. 28) and the results, as derivable by the facility of De Moivre's Hypothesis. Age twenty (Dr. Halley's method), £12.78. (De Moivre's Hypothesis,) £12.30. Age thirty, (Dr. H.'s) £11.72. (De M.'s) £11.61. Age forty, £10.57 & £10.70. Age fifty, £9.21 & 9.49. Age sixty, £7.60 & 7.83. Age seventy, 5.32 & 5.50.

It will thus be seen, that although in the pre-

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* Some limit of a similar kind has generally been fixed in all such calculations:

Thus, in the " Northampton" table the limit was originally assumed as 93 (*vide* Dr. Price's Observ. first edit. 1771. p. 317), but afterwards extended to 97 (*ibid.* fourth edit. 1783, vol. II. p. 36).—In the " Carlisle" table the limit is 105 (*vide* Milne's Treat. vol. II. p. 564).—In the " Equitable Experience " 98 otherwise 99 (*vide* Davies' Treat. Tab. X. & Ar· Morgan's Tab. p. 28 ; also Babbage's Treat. Tab. VIII).—" Amicable Experience" 99 (*vide* Galloway's Tab. p. 34).—" Combined Experience of Seventeen Life Assurance Offices " 100 (*vide* Committee's Report, Table (G), p.41).—" Chester," Males 96, Females 98 (*vide* Price Obs. 1783. vol. II. p. 113).—" Government Annuitants," Males 99, " among the 22,352 Nominees registered by me there was " only one instance of a person passing the age of 98; an old Lady at " Wimbledon, who lived to be 100 years old. (*vide* Finlaison's Report, p. 19, and Tab. 58).

† De Moivre's Hypothesis was evidently capable of representing an approximation to any other Table of Mortality, by equating the annual decrement to $\frac{1}{w}$th part of the representative number born.

sent advanced state of Vital Statistics, such values would not be deemed sufficiently approximative, yet, at the period of Dr. Moivre's publication, when even distinctions of age were practically overlooked, the differences between the two series were relatively too trifling to be worthy of consideration.

De Moivre, after fully discussing the application of his hypothesis to the various cases involving the contingency of a single Life, proceeds to the consideration of " several lives." In the solution of such problems, he proposes a new theory, constituting a geometric in place of an arithmetic progression in the probabilities of Life, but as such secondary theory did not afterwards become current, further notice of it appears unnecessary.

In the Appendix alluded to in his title page, " concerning the Expectations of Life and Probabilities of Survivor- " ship," De Moivre considers the application of his Hypothesis to the subject of Life-measurement, without relation to financial values.

Among the questions he proposed for exemplification, was one which in principle, has since been generally adopted in Treatises on Life-contingencies, and constitutes what is called " the Expectation of Life." De Moivre's proposition was, " To find how " long it may be expected that a Life of a given Age " may continue in being." (p. 80.) In connection with his hypothesis he resolves this question, by means of the following theoretical rule : " If it be requi- " red to find the Expectation of a Life of which the age is given; " the rule for finding it is expressed in these few words; take half " the difference between the age given, and the Extremity of Old " Age, supposed at 86, and that half difference is the thing required; " thus, if the age be 50, then the difference between 50 and 86

" being 36, whereof the half is 18; it may be concluded that 18 years
" is the duration which a Life of 50 may justly expect." (De
Moivre, p. v.)*

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* The equality of the *average* and *probable* durations of life when
the annual decrement is considered uniform (as by De Moivre's
Hypothesis) probably occasioned their being subsequently confounded.

That De Moivre, however, of these two significations meant the
term Expectation to express *average* duration, is obvious, from his
saying, " *no just estimate can be made of the Expectation of Life, but*
" *as it is compared with an Annuity, of which the purchase admits of*
" *no discount, and to which there is a payment due, even to the last*
" *moment of Life.*" (p. vi.)

The additional half-year, commonly recognised in such calcula-
tions, was first proposed by De Moivre; as appears by the following
Corollary, occurring in illustration of the proceeding that should be
adopted, if the annual decrement were supposed to change periodi-
cally at particular intervals; it being, however, necessary to premise
that De Moivre, in his subsequent example, includes the initial num-
ber of the first interval. " *If the Complement of Life be divided into*
" *equal intervals, and we take out of the Table of Observations the*
" *Numbers terminating those intervals, and from the Sum of all those*
" *Numbers, we subtract one-half of the First, and then multiply the*
" *Remainder by One of the equal Parts, and lastly divide the Product*
" *by the Number answering to the Beginning of the First interval; the*
" *Quotient will express the Number of Years which compose the Expec-*
" *tation of the Life given.*" (p. 82.)

From which it is evident that if the equal intervals were presumed
to be annual, the resulting formula in modern notation would be,

$$\frac{(a_1 + a_2 + a_3 + a \text{----} + \omega - \frac{a}{2}) \times 1}{a}$$ which is obviously equal to

$(a_1 + a_2 + a_3 \text{ ad ætat: ult:}) + \frac{1}{2}$, the expression now currently employed.

De Moivre seems to have derived " Expectation," as a *word* from
his previous Doctrine of Chances, " The Expectation of obtaining
" any thing is estimated by the value of that thing multiplied by
" the Probability of obtaining it." (Doctrine of Chances, 1718, p. 2.)
" To find the Expectations of the Gamesters, each term of the series
" expressing these probabilities, is to be, &c." (*ibid.* p. 92.)

It has been commonly considered that De Moivre is only to be recollected, in the History of Life-contingencies, as the proposer of the Theory bearing his name, and that as this Theory has long since fallen into disuse, that his labours have proved altogether nugatory. But historical justice requires that each Treatise should be judged in relation to its date; estimated by this comparison De Moivre's forms undeniably to a reader conversant with such matters, the most original work yet extant, and must not only have very considerably aided the primary *discussion*, and therefore improvement of the subject, but has moreover incidentally afforded even in the present day, one of the only two correct Annuity formula extant.

[1726] In 1726, was published the quarto edition of the well-known Interest Tables, "by John Smart," to which he appended, a few remarks " Of Annuities upon Lives."

" I did intend to have calculated some Tables, of the probable
" Values of One, Two and Three Lives, according to the different
" Ages of Mankind, and at several Rates of Interest; But, as such
" Tables must be founded upon Observations made upon the Number
" of Persons dying at such different Ages, and as I could meet with
" very few such Observations that could be depended on, and those
" but for a few Years, I laid aside my Design; not doubting but here-
" after, when proper observations shall have been made, some one or
" other will take the Pains of constructing such Tables.

" In order thereunto, if those in whose Power it is, would oblige
" the Parish Clerks weekly to make a Return of the Age of every
" Person dying in each respective Parish within the weekly Bills of
" Mortality, in like manner as they now make their Returns of the
" Diseases and Casualties; and if the Number dying of every Age
" were printed at the end of every Year, with the yearly Bill; then
" there would in time be a good Foundation to build upon: and when
" ever this is done in London, it is probable it will be done likewise

" in other Places both at home and abroad, where Accounts of Births
" and Burials are kept, which would make the Foundation still more
" certain." (Smart's Tables, 1726, p. 113)*

[1727] In 1727, a treatise appeared, entitled,
" A new method for Valuing of Annuities upon Lives, &c., by Ri-
" chard Hayes." Mr. Hayes' work, is merely of an
arithmetical and popular character, but appears
to be the first publication in which a Table of
Life Annuities at yearly ages was inserted (30 to
73); for though Dr. Halley and De Moivre had
both explained the methods of proceeding, yet
neither had supplied otherwise than a specimen
Table

Mr. Hayes does not state how he computed his
Tables otherwise than by saying at the commence-
ment, that he had calculated them " according to
" the chance of Annuitants living to the extremity of
" the common oldest age of Life," and this upon
suppositions of " the various degrees of probability
" which Lives of different ages have to continue in
" being." As, however, this last description also
occurs in the Preface to De Moivre's Treatise, it is

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* As Mr. Smart's Tables are still currently employed for Interest
purposes, it may be well to incidentally remark that the half-yearly
values should be rejected as merely theoretical.

Mr. Smart calculated his Tables in accordance with the prevalent
notion, that a sum of money at interest increased in geometric
proportion between the *ordinary* yearly values: by which supposition
the paradox was indirectly incurred, that for fractional intervals
Compound would be less remunerative than Simple interest. Thus
if $(1 + r)$ represent the amount in one year, the factor for the
interval $\frac{1}{n}$ would be $(1 + r)^{\frac{1}{n}}$, which is evidently less than $(1 + \frac{r}{n})$,
the simple-interest coefficient.

probable that Mr. Hayes adopted some arithmetical version of the Hypothesis originated by that author.

The popular character of Mr. Hayes's work will be sufficiently illustrated by the following example:

" To PROVIDE FOR A FAMILY.—A Clergyman, or Layman, aged
" 47 Years, holding a Benefice or Place during Life, and having a
" Family, would willingly make some certain Provision for them; but
" finding that his Income will let him lay up about 46l. a Year, and
" that upon no better Security than his own uncertain Life, therefore
" chuses rather to sell the Surplusage of his Income. The Question
" is, What Provision the said 46l. a Year will make for his Family,
" admitting the money is valued at the common legal Rate of 5 ⅌
" Cent. Interest.

$$*\qquad*\qquad*\qquad*\qquad*\qquad*$$

" The Value of this Annuity* to the Chance of Age 47, is worth
" upon that Life at 5 ⅌ Cent. as may be seen in the Tables at the
" latter Part of this Book, showing the Value of Annuities upon Life,
" as follows:—

	l.	s.	d.	
" 40l. a Year is worth	418	0	11	} added.
" 6 ——— ———	62	14	1	
" Shows 46l. a Year is worth	480	15	0	certain for his Family."

(Hayes' Annuities, 1727, p. 7.)

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* If we temporarily suppose Life-Assurance formulæ to have been known at this date, and Establishments to have existed, admitting distinctions of age, then, by merely rearranging Mr. Hayes' *own data* for a *Life-Assurance*, in lieu of an *Annuity* transaction, £46 per annum surrendered during the Life-time of a person aged forty-seven, would have secured £1057 0 10, as the sum assured.

For when the Annuity is a nomial part, $(1+A)^{-1} + (1+r)^{-1} - 1$, represents the *premial*-payment, consequently the *Annuity*-payment would be in the ratio to such of $1+A : A$, whence the factor would become $A^{-1}(1+A)$, and the product, $A^{-1}(1+A)(1+r)^{-1} - 1$. By numerical substitution, $r = 0.05$ & $A = 10.45$, then the unital ratio would be 0.043518, whose reciprocal multiplied by 46 equals 1057.039 or £1057 0 10, as above specified.

[1728.] In the following year (1728) Mr. Hayes published " An Estimate of places for Life, showing how many " years' purchase a Place for Life is worth, &c." This work, like the former, is merely a popular book of examples, yet, by pourtraying that the value of Life-Incomes ought to be estimated, in each case, according to the age of the possessors, probably also, more or less tended to popularize among a certain class of readers, the Doctrine of specific valuations.

In page 53, were inserted some remarks, as offered by Mr. Smart in an Appendix to his Interest Tables, advising the insertion of ages in the London Bills of Mortality; apparently, in pursuance of this recommendation, such details were added for the first time, in 1727-8.

The following was the result of the first year's " Bill of Mortality with ages." (Dec. 11th, 1727, to Dec. 10th, 1728):

	Males	.	8497			Males	.	13538
" Christened, *	Females	.	8155	Buried,		Females	.	14272
	In all	.	16652			In all	.	27810

" Decreased in the Burials this Year, 698.

" Whereof have died,

Under 2 Yrs. of Age	9851	20 and 30	2254	60 and 70	. .	1863
Between 2 and 5 . .	2407	30 and 40	2490	70 and 80	. .	1290
5 and 10	1038	40 and 50	2624	80 and 90	. .	785
10 and 20	950	50 and 60	2123	90 and upwards		135

* It may be well to warn the general reader not to interpret the word " Christened," as synonymous with " Born." The former merely representing such a proportion of the Births as were registered in connection with the rites of the Established Church. This observation appears necessary, lest it might be supposed that as the number specified above as buried, exceeded the number christened, that the City was gradually being depopulated; an inference which misled several authors at a subsequent period.

Thk "Bills of Mortality" having been frequently noticed in the preceding pages, the writer thinks it expedient to here insert the general Bill for the year Dec. 1727-8, being the date at which ages were first included. The Bill thus presented will also illustrate the character of those of an earlier date, the chief differences being those already detailed in pages 11 and 12, with an occasional addition to the number of Parishes comprised, and trifling modifications in the List of Diseases and Casualties.

A GENERAL BILL OF ALL THE CHRISTENINGS AND BURIALS,

From the 11th December, 1727, to the 10th of December, 1728.

According to the Report made to the King's most excellent Majesty, by the Company of Parish Clerks of London, &c.

Parish	Bur.	Parish	Bur.	Parish	Bur.
St. Alban in Wood-street	27	St. Gregory by St. Paul's	60	St. Mary Mounthaw	9
Alhallows Barkin	70	St. Helen near Bishopsgate	25	St. Mary Somerset	41
Alhallows in Bread-street	13	St. James in Duke's Place	31	St. Mary Staining	17
Alhallows the Great	61	St. James at Garlickhithe	25	St. Mary Woolchurch	0
Alhallows in Honey-lane	0	St. John Baptist by Dowgate	31	St. Mary Woolnoth	25
Alhallows the Less	25	St. John the Evangelist	6	St. Matthew in Friday-street	3
Alhallows in Lombard-street	21	St. John Zachary	21	St. Michael Bassishaw	52
Alhallows Staining	19	St. Katherine Coleman	69	St. Michael in Cornhill	14
Alhallows on London Wall	40	St. Katherine Creechurch	79	St. Michael in Crooked-lane	41
St. Alphage near Sion College	39	St. Laurence Jewry	42	St. Michael at Queenhithe	30
St. Andrew Hubbard	14	St. Laurence Pountney	14	St. Michael at Quern	17
St. Andrew Undershaft	39	St. Leonard in Eastcheap	12	St. Michael Royal	23
St. Andrew Wardrobe	67	St. Leonard in Foster-lane	31	St. Michael in Wood-street	13
St. Ann within Aldersgate	36	St. Magnus by London Bridge	48	St. Mildred in Bread-street	19
St. Ann in Black Friars	167	St. Margaret in Lothbury	19	St. Mildred in the Poultry	35
St. Anthony, vulg. Antholin	5	St. Margaret Moses	6	St. Nicholas Acons	3
St. Augustin, vulg. Austin	22	St. Margaret in New Fish-str.	0	St. Nicholas Coleabby	24
St. Bartholomew by Exchange	21	St. Margaret Pattons	16	St. Nicholas Olave	6
St. Benedict, v. Bennet Fink	17	St. Martin Ironmonger-lane	14	St. Olave in Hart-street	54
St. Bennet Gracechurch	17	St. Martin at Ludgate	30	St. Olave in the Old Jewry	4
St. Bennet at Paul's Wharf	38	St. Martin Orgars	22	St. Olave in Silver-street	50
St. Bennet Sherehog	1	St. Martin Outwich	20	St. Pancras in Pancras-lane	1
St. Botolph at Billinsgate	5	St. Martin Vintrey	35	St. Peter in Cheapside	4
Christ Church Parish	185	St. Mary Abchurch	24	St. Peter in Cornhill	49
St. Christopher's Parish	21	St. Mary Aldermanbury	37	St. Peter near Paul's Wharf	15
St. Clement near Eastcheap	20	St. Mary Aldermary	26	St. Peter Poor in Broad-street	24
St. Dionis Backchurch	30	St. Mary le Bow in Cheapside	31	St. Stephen in Coleman-street	48
St. Dunstan in the East	80	St. Mary Bothaw at Dowgate	5	St. Stephen in Walbrook	16
St. Edmund the King	21	St. Mary Colechurch	0	St. Swithin at London-stone	23
St. Ethelburga's Parish	30	St. Mary Hill by Billinsgate	37	St. Thomas the Apostle	11
St. Faith under St. Paul's	39	St. Mary Magd. in Milk-street	0	Trinity Parish	31
St. Gabriel in Fenchurch-str.	20	St. Mary Magd. Old Fish-str.	29	St. Vedast, alias Foster	28
St. George in Botolph-lane	16				

Christened in the ninety-seven Parishes within the Walls, 1574. Buried, 2793.

Parish	Bur.	Parish	Bur.	Parish	Bur.
St. Andrew in Holborn	1245	Bridewell Precinct	35	St. Saviour in Southwark	888
St. Bartholomew the Great	45	St. Bridget, vulg. St. Bride's	387	St. Sepulchre's Parish	784
St. Bartholomew the Less	35	St. Dunstan in the West	262	St. Thomas in Southwark	371
St. Botolph by Aldersgate	248	St. George in Southwark	592	Trinity in the Minories	34
St. Botolph by Aldgate	833	St. Giles by Cripplegate	1694	The Pest-house	
St. Botolph by Bishopsgate	564	St. Olave in Southwark	877		

Christened in the sixteen Parishes without the Walls, 5570. Buried, 8894.

Parish	Bur.	Parish	Bur.	Parish	Bur.
Christ Church in Surrey	213	St. John at Wapping	324	St. Mary Magd. Bermondsey	516
St. Dunstan in Stepney	2812	St. Katherine near the Tower	198	St. Mary at Newington	382
St. Giles in the Fields	1481	St. Leonard in Shoreditch	858	St. Mary at Rotherhith	310
St. James at Clerkenwell	621	St. Mary at Islington	269	St. Mary in Whitechapel	1085
St. John at Hackney	217	St. Mary at Lambeth	553	St. Paul at Shadwell	623

Christened in the fifteen Out Parishes in Middlesex and Surrey, 6027. Buried, 10462.

Parish	Bur.	Parish	Bur.	Parish	Bur.
St. Ann in Westminster	463	St. Margaret in Westminst.	1145	The precinct of the Savoy	73
St. Clement Danes	582	St. Martin in the Fields	1600	St. Paul in Covent Garden	283
St. James in Westminster	1409	St. Mary le Strand	106		

Christened in the eight Parishes in the City and Liberties of Westminster, 3481. Buried, 5661.

DISEASES AND CASUALTIES.

Disease	No.	Disease	No.	Disease	No.	Casualties	No.
Abortive	55	Gravel	4	Rickets	103		
Aged	2768	Grief	13	Rising of the Lights	41	CASUALTIES.	
Ague	44	Griping in the Guts	433	Rupture	12		
Apoplexy	73	Headmouldshot	40	St. Anthony's Fire	6	Broken Limbs	2
Asthma	156	Hooping Cough	21	Scald Head	5	Bruised	3
Bedridden	14	Horshoehead	28	Scurvy	3	Burnt	2
Bleeding	3	Jaundies	132	Small Pox	2105	Died for Want	6
Bloody Flux	23	Imposthume	28	Sores and Ulcers	36	Died by the Bite of a mad Dog	1
Cancer	49	Inflammation	4	Spleen	3		
Canker	16	Itch	3	Spotted Fever	94	Drowned	89
Chicken Pox	1	Leprosy	8	Stillborn	533	Excessive Drinking	22
Childbed	216	Lethargy	13	Stone	43	Executed	37
Colick	78	Livergrown	10	Stoppage in Stomach	147	Found Dead	44
Concussion of the Brain	1	Loosness	33	Strangury	4	Frighted	2
Consumption	3491	Lunatick	26	Suddenly	101	Killed by Falls and other Accidents	44
Convulsion	7517	Malignant Fever	2	Surfeit	8		
Cough	5	Measles	82	Swelling	1	Made away themselves	59
Diabetes	3	Miscarriage	1	Teeth	1676	Murdered	6
Dropsy	1125	Mortification	238	Thrush	84	Overlaid	71
Evil	35	Palsy	28	Tissick	456	Poisoned	1
Fever	4716	Pleurisy	48	Twisting of the Guts	16	Scalded	4
Fistula	10	Purples	10	Tympany	18	Stabbed	1
Flux	14	Quinsy	11	Vomiting	6		
French Pox	113	Rash	2	Water in the Head	73	Total 397	
Gout	39	Rheumatism	25	Worms	31		

CHRISTENED,	Males 8497	BURIED,	Males 13538
	Females 8155		Females 14272
	In all 16652		In all 27810

Decreased in the Burials this Year, 608.

Whereof have died,

Under Two Years of Age	9851	Twenty and Thirty	2254	Sixty and Seventy	1863
Between Two and Five	2407	Thirty and Forty	2490	Seventy and Eighty	1290
Five and Ten	1038	Forty and Fifty	2624	Eighty and Ninety	785
Ten and Twenty	950	Fifty and Sixty	2123	Ninety and upwards	135

[1730] In 1730, was published the first work that contained Tables for " two and three Lives;" the title was as follows: " The Gentleman's Steward and " Tenants of Manors instructed, &c. The tables for valuing Es- " tates on Lives being founded on Dr. Halley's Hypothesis, and " calculated by the method laid down by Mr. Abr. De Moivre, to " 4, 5, 6, 7, and 8 per Cent. &c. by John Richards, of Exon."

Mr. Richards, though merely a popular writer, and apparently not more than ordinarily skilled in lease matters, had the acuteness to perceive that the true method of " valuing leases for Lives" was really dependent upon specific calculation, and that the imaginary estimates currently adopted by " Stewards and Tenants of Manors" frequently proved disadvantageous to their own interests, even when the contrary was primarily presumed.

To supply proper Tables, &c. for such purposes, appears therefore, to have been the design of the above publication.

" I don't know of any thing (like a rational Scheme) as yet pub- " lished for adjusting the Values of Leases on the several Tenures in " Use amongst us; one Reason of which may be perhaps the Diffi- " culty, or rather the seeming Perplexities with which such a Com- " putation is involv'd. For although several persons have, with good " Success endeavour'd to obviate Part of the Difficulties; yet the " Application of their Precepts to the Purposes before-mention'd, " does still remain a Task insuperable to the Generality of People " whom they concern; the Gentleman's Steward and Tenants being " (for the most part of them at least) as much in the dark as ever.

" And indeed as the Foundation of such a Scheme was too noble " to be laid by a less Genius than that of the great Dr. Halley, or " Mr. De Moivre; so likewise are there some porterly Offices neces- " sary in carrying on the Structure, (such as making the following " Tables, &c.) that are too mean, too ignoble a Drudgery for the " World to expect from their Labours." (p. xxv.)

Accordingly in this self-styled " porterly," but

really laudable spirit, Mr. Richards, to remedy the existing want of Life-Annuity Tables, had the industry to calculate

" The Worth of an Annuity that is to continue 7, 10, 14, or 21
" Years, if a Person of a given Age do live so long; and these are
" calculated by the Method laid down by Mr. Abr. de Moivre, in his
" Annuities on Lives, Prob. 2. pag. 21."

" The Value of an Annuity that is to continue during the Life of a
" Person of any Age, at 4, 5, 6, 7, and 8 per Cent."

" The Value of two joint Lives of equal Ages. By joint Lives is
" meant, that the Annuity is to continue till one of those Lives is
" dead, and no longer."

" The Value of two joint Lives of different Ages.

" The Value of three joint Lives of different Ages.

" All these five last-mentioned Tables are built on the Foundation
" laid by the great Dr. Halley, in Philosophical Transactions, No. 196.
" and the Calculus perform'd by the Method set forth by Mr. De
" Moivre in his Tract before-mentioned."

" Having gone thro' with these Tables, (and pretty well sur-
" feited myself on Figures)" he desisted, " for above three Years;
" during which Interval, that Surfeit of making Tables was pretty
" well worn off; and considering that it would render the Work more
" useful, if Tables of the Value of an Annuity on the longest Liver
" of two or three Persons were subjoined to it: I have with a pretty
" deal of trouble perfected such Tables; whereby the Value on such
" Tenure (which is the most in Use of any amongst us) may be found
" for Lives of any Ages, howsoever combined, at one View." (*ibid.*
p. xi-xiv.)

The following commencing portions of two of the Tables will sufficiently indicate to the reader conversant with such matters, the laborious nature of the remainder of the task; to other readers it may be sufficient to state that these Tables for " two and three lives " published by Mr. Richards, though constituting the first attempt of the kind, occupy several octavo pages, and include 2100 Tabular values.

Table VI.

"Shewing the Value of Annuities for two joint Lives, &c."

One Life 2 Years old.

Age.	4 ⅌ Cen.	5 ⅌ Cen.	6 ⅌ Cen.	7 ⅌ Cen.	8 ⅌ Cen.
2	10.89	10.04	9.24	8.50	7.84
12	11.88	10.99	10.11	9.28	8.48
22	11.28	10.40	9.60	8.87	8.21
31	10.37	9.68	9.01	8.35	7.67
42	9.36	8.77	8.19	7.62	7.07
52	8.17	7.74	7.31	6.88	6.45
62	6.59	6.29	5.99	5.70	5.41
72	4.52	4.37	4.22	4.07	3.93
82	1.33	1.31	1.29	1.27	1.25

One Life 12 Years old.

Age.	4 ⅌ Cen.	5 ⅌ Cen.	6 ⅌ Cen.	7 ⅌ Cen.	8 ⅌ Cen.
12	13.01	12.05	11.13	10,22	9.32
22	12.37	11.41	11.50	9.67	8.88
32	11.27	10.96	9.84	9.11	8.39
42	10.52	9.47	8.84	8.22	7.64
52	8.73	8.89	7.84	7.38	6.91
62	6.94	6.68	6.39	6.08	5.75
72	4.71	4.57	4.42	4.26	4.10
82	1.35	1.34	1.32	1,30	1.28

Table VII.

"Wherein is shewn the Value of an Annuity for three joint Lives."

One Life 2 Years old. (2d Life / 3d Life — Two Years old.)

3d Life	4 ⅌ C.	5 ⅌ C.	6 ⅌ C.	7 ⅌ C.	8 ⅌ C.
2	8.44	7.97	7.49	7.01	6.53
12	9.03	8.60	8.11	7.60	7.66
22	8.69	8.21	7.73	7.26	6.80
32	8.17	7.87	7.50	7.09	6.44
42	7.49	7.25	6.90	6.52	6.04
52	6.69	6,44	6.13	5.85	5.54
62	5.55	5.31	5.06	4.92	4.75
72	3.98	3.89	3.78	3.67	3.53
82	1.25	1.23	1.21	1.20	1.18

One Life 12 Years old. (Twelve Years old.)

	4 ⅌ C.	5 ⅌ C.	6 ⅌ C.	7 ⅌ C.	8 ⅌ C.
12	9.70	9.23	8.75	8.21	7.58
22	9.31	8.85	8.37	7.85	7.30
32	8.66	8.28	7.89	7.43	6.92
42	7.94	7.60	7.24	6.86	6.46
52	7.04	6.79	6 50	6.28	5.88
62	5.82	5.65	5.46	5.24	4.98
72	4.12	4.04	3.93	3.82	3.68
82	1.28	1.27	1.27	1.26	1.25

(Richard's "Gentlemen's Steward, &c." 1730. pp. 38-41.)*

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* Mr. Richard's Tables include all the interchanges of the above nine specified ages, both for "two" and "three Lives," and for "joint Lives," and "survivors;" consequently, the total number of values may be represented by

$$2 \left(n + \frac{n(n-1)}{1 . 2} \right) + 2 \left(n + n(n-1) + \frac{n(n-1)(n-2)}{1 . 2 . 3} \right) = 420$$

at each of the five rates of interest, or 2100 in total, as specified above.

For a similar but complete modern series by annual intervals for "two" and "three Lives," and for "joint Lives," and "survivors," the expression would become $\omega = 105$, (Carlisle limit, as noticed in p. 50,) then by substituting ω for n

$$2 \left(\omega + \frac{\omega(\omega-1)}{1 . 2} \right) + 2 \left(\omega + \omega(\omega-1) + \frac{\omega(\omega-1)(\omega-2)}{1 . 2 . 3} \right) =$$

408,100 values, in connection with each rate of interest, or in total, 2,040,500 Tabular *results*.

In the same year (1730) was published, "A Dissertation on Estates upon Lives, &c. by Edwd. Laurence, Land-Surveyor," in which Dr. Halley's Table was inserted, and recommended for adoption, in preference to the imaginary estimates on Lives then current. In other respects, Mr. Laurence's Dissertation, being merely discursive, does not require comment.

[1735] In a similar manner it will suffice to mention, that in 1735, a little book appeared, called "Tables for Renewing and Purchasing of Leases, and also for Renewing and Purchasing of Lives, &c. by Gael Morris." It includes Dr. Halley's Table at 4 per Cent. by approximation, and popularly illustrates the application to Life-leases, &c. of four of De Moivre's problems.

These and other minor publications indirectly illustrate to us, that the Doctrine of Life-valuation was gradually extending towards those transactions, in which its utility, was really most capable of being made available.

[1737] In 1737, was published anonymously "An Essay to ascertain the value of Leases of Annuities for Years and Lives, and to Estimate the Chances of the Duration of Lives." In the following year the work was re-printed, and the name of Weyman Lee added to the title page.

The idea upon which Mr. Lee based his estimates, though inherently false, was probably extremely plausible to popular readers. Mr. Lee stated that a Life-Annuity at any particular age, ought to be considered as exactly equal in value, to an Annuity-certain for as many years as the life had an equal chance of surviving.

Thus, if it were an equal chance that a person of a particular age would live fifteen years, then

Mr. Lee contended that a Life-Annuity for such age, ought to be equal in value to an Annuity-certain for fifteen years, because as an equal number of such persons would die before and after that term, the medial limit should represent the average measure.

Finding that Dr. Halley's and De Moivre's calculations were not conformable to this dictum, Mr. Lee attempted to prove that the mathematical principles of those authors were untenable; but from Mr. Lee's not possessing mathematical acquirements, and thus not comprehending mathematical demonstrations, &c. the reader will not be surprised to learn that in the 340 octavo pages devoted to the topic, (p.p. 130-470) Mr. Lee rather exemplified the absurdity of his own pretensions, than weakened the arguments of the celebrated mathematicians he had presumed to attack.

To the modern reader, the inconsistency of comparing Life-Annuities with any description of Annuities-certain, is too obvious to need exemplification.

[1738] In 1738, a Second Edition of De Moivre's "Doctrine of Chances" being published, he inserted, as a species of appendix, some remarks "Of Annuities on Lives," recapitulating the chief substance of his separate Treatise on the subject, with a few additional problems. He also incidentally offered a formula which, in another form, now constitutes one of the most important elements in such calculations. Its purport was to define the prospective value, of a sum to be receivable, if a person should die within a specified number of years.

This problem De Moivre demonstrated was resolvable by a similar method of annual values, to that proposed by Dr. Halley for Life-Annuities.

Thus, the chance of the sum's becoming payable during the first year was to be estimated in relation to the chance of death in that year; the chance of receiving the sum in the second year, by the chance of death in such year; and so respectively proceeding for each future year. The addition of all these separate values affected by the relative discounts would then form the total value of the expectant reversion in precisely the manner that the total value of a Life-Annuity was originally ascertainable.*

De Moivre, on this occasion, also inserted a Table in accordance with his Hypothesis, of Life-Annuity values, at 5 per cent. interest, for every year of age, from one to eighty-five inclusive, and thus appears to have offered the first *complete* Table extant.

In this year also, Mr. Smart, who had originally advised the insertion of ages in the London Bills,

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* De Moivre's own formula was

$$\frac{A-B}{Ar} + \frac{B-C}{Arr} + \frac{C-D}{Ar^3} + \frac{D-E}{Ar^4} + \frac{E-F}{Ar^5} \&c.$$

(Doct. of Chan. 2nd Edit. 1738, p. 212), in which expression may be recognised the primary elements of Col. M. in modern calculations.

It may be here allowable to incidentally notice that the original framing of the *Column* system of calculation referred to, has generally been attributed to the late Mr. Barrett, whose method was published in the form of an Appendix to Mr. Baily's Treatise on Annuities as reissued in 1813. That we are principally indebted to this source for the prevalent adoption of the system is undoubted, but a prior embodiment of it, will be found in the Treatise of the late Mr. W. Morgan, published as early as the year 1779. The reader generally conversant with such matters, will at once admit the correctness of this assertion, by consulting p. 64 of Mr. Morgan's Treatise, in which will be found a Table of the following form, which though primarily offered merely for the purposes of rectification, has nearly all the inherent qualities of modern arrangements —

(vide p. 56), published a Table of Mortality, founded on the first ten years' results. As this Table of Mr. Smart's was the first composed from English data, it is here inserted, preceded by the returns upon which it was founded:

FIRST TABLE OF THE VALUES OF SINGLE LIVES.

Age.	Values of Lives.	Values of £1 payable, &c.	Sums.
92	0000000	.0000332	0000000
91	.480769	.0000690	.0000332
90	.711908	.0001436	.0001022
89	1.097377	.0002244	.0002458
88	1.512531	.0003106	.0004698
87	1.93271	.0004038	.0007804
86	2.16983	.0005460	.0011842
&c.	&c.	&c.	&c.

Thus, the nature of the third column being represented (modern notation) by $_n av^n$ that of the fourth may be considered as $S \left(_{n+1} av^{n+1} \right)$ consequently the value of complete and temporary Life-annuities, &c. would be nearly as readily determinable by these means, as if the columns had represented the values of the numerators $^n av^n$ & $S \left(^n av^n \right)$ as at present in use. (Cols. D & N.)

In page 70 the Tabular values are adapted to the common denominator, 1000, &c.; and in page 74, the system is extended to Joint Lives; Mr. Morgan concluding the chapter by observing, " as by " these methods the calculations are rendered pleasant as well as " expeditious, I hope that ere long some person will undertake them, " chusing for his guide the Northampton Table of Observations, " which perhaps is better fitted for common use than any other." Mr. Barrett appears to have pursued this recommendation, and to have incidentally detected that the system was capable of further extension and uses, though his unnecessary adoption of the reversal of ages, clearly indicates Mr. Morgan's Treatise to have been the suggestive source. For the re-arrangement now current, and the addition of a Decremental Series, (Col. M,) we are indebted to the discrimination of Mr. Griffith Davies as instanced by his Treatise on Life-Contingencies, (1825,) of which the system referred to, forms the leading characteristic.

Table of the Burials in the City and Suburbs of London during the first Ten Years in which the Ages were inserted in the Bills of Mortality. (Dec. 11th, 1727, to Dec. 13th, 1737.)

AGES OF PERSONS.	Buried. 1728	Buried. 1729	Buried. 1730	Buried. 1731	Buried. 1732	Buried. 1733	Buried. 1734	Buried. 1735	Buried. 1736	Buried. 1737	Numb. Total.
Under Two Years of Age	9851	10735	10368	9907	9502	11738	10752	9672	10580	10054	103159
Between Two and Five ..	2407	2516	2448	2096	1517	2409	2830	1963	2706	2613	23505
Five and Ten	1038	1056	1092	932	716	957	1228	755	993	1008	9775
Ten and Twenty . ..	950	999	901	806	611	754	829	691	816	885	8242
Twenty and Thirty ..	2254	2371	2048	1916	1627	1857	1718	1605	2139	2241	19776
Thirty and Forty ..	2490	2784	2471	2351	2175	2564	2212	2158	2445	2652	24302
Forty and Fifty ..	2624	2698	2373	2261	2121	2685	2154	2138	2357	2578	23989
Fifty and Sixty ..	2123	2338	1713	1839	1741	2196	1668	1684	2121	2270	19693
Sixty and Seventy ..	1863	1938	1577	1500	1581	1871	1324	1339	1666	1650	16309
Seventy and Eighty..	1290	1375	1001	913	974	1188	793	872	1114	1164	10684
Eighty and Ninety ..	785	769	622	628	660	804	484	565	557	576	5750
Ninety and upwards..	135	143	147	113	133	210	70	96	87	132	1266
Numb. total ..	27810	29722	26761	25262	23358	29233	26062	23538	27581	27823	266450

Ages.	Number of Persons.	Die off Yearly.	Ages.	Number of Persons.	Die off Yearly.	Ages.	Number of Persons.	Die off Yearly.	Ages.	Number of Persons.	Die off Yearly.	Ages.	Number of Persons.	Die off Yearly.
Born	1000	290	20	459	6	40	294	10	60	130	7	80	29	3
1	710	96	21	453	6	41	284	10	61	123	6	81	26	3
2	614	50	22	447	7	42	274	10	62	117	6	82	23	2
3	564	25	23	440	7	43	264	9	63	111	6	83	20	3
4	539	13	24	433	7	44	255	9	64	105	6	84	17	3
5	526	10	25	426	8	45	246	9	65	99	6	85	14	2
6	516	8	26	418	8	46	237	9	66	93	6	86	12	2
7	508	7	27	410	8	47	228	8	67	87	6	87	10	2
8	501	6	28	402	8	48	220	8	68	81	6	88	8	2
9	495	5	29	394	9	49	212	8	69	75	6	89	6	1
10	490	4	30	385	9	50	204	8	70	69	5	90	5	1
11	486	4	31	376	9	51	196	8	71	64	5	91	4	1
12	482	3	32	367	9	52	188	8	72	59	5	92	3	1
13	479	2	33	358	9	53	180	8	73	54	5	93	2	1
14	477	2	34	349	9	54	172	7	74	49	4	94	1	1
15	475	2	35	340	9	55	165	7	75	45	4	95	0	
16	473	2	36	331	9	56	158	7	76	41	3			
17	471	3	37	322	9	57	151	7	77	38	3			
18	468	4	38	313	9	58	144	7	78	35	3			
19	464	5	39	304	10	59	137	7	79	32	3			

It has been necessary, on account of the original size, to alter the tabular form of Mr. Smart's table in the present instance. The general reader, on referring to the account of Dr. Halley's Table, (p. 25,) will easily apprehend the application of the table now offered.

[1739.] In 1739 was published a Treatise entitled " Annuities for Lives and for Limited Terms of Years considered, &c. " by John Richards."

Mr. Richards' former work (vide p. 59,) having been animadverted on by Mr. Weyman Lee, Mr. Richards appears to have issued the above named publication merely as a species of reply in defence of the methods of Dr. Halley and De Moivre he had originally adopted. The absurdity of the principle advanced in Mr. Lee's Essay having already been noticed, (vide p. 62,) Mr. Richard's reply will not require further comment.

[1742.] In 1742 appeared " The Doctrine of Annuities " and Reversions deduced from general and evident principles, with " useful Tables, shewing the values of Single and Joint Lives, &c. " by Thomas Simpson."

Mr. Simpson was already well known by his " Treatise on Fluxions," " Nature and Laws of Chance," and his Essays, as a superior mathematician, and thus his name alone would have been an important accession in aid of the general acceptance of Life-measurement as a subject of definite calculation.

The same topic, however, that had already been considered worthy of consideration by the celebrated Dr. Halley and De Moivre was still more appreciated by Simpson. He seems, with natural acuteness, to have foreseen that the Doctrine of Life-contingencies was destined, at a future time, to be extensively employed, and that consequently, more real utility would result from endeavouring to discover general demonstrations applicable to *all* tables of Observations that might be produced from time to time, than from inventing particular hypotheses, which however interpretative of cotemporary data, might cease to be useful if new data should arise. " What, I apprehend, may

" best recommend this performance, is the general, yet familiar man-
" ner in which the subject is treated, there not being a Solution
" throughout the whole work, except those relating to the use of the
" tables, that is not universal, according to any table of observations
" or degree of probability of life whatsoever; and yet the conclusions
" and practical rules deduced therefrom, are, for the most part, alto-
" gether as simple, as could be derived from any hypothesis." (p. vi.)*

In illustration of the particular Table of Mortality

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* The reader will perceive by the above extract, that Simpson vir-
tually declared in contravention of De Moivre's method of pro-
ceeding, (vide p, 48,) and collaterally reverted to the original pro-
cedure as proffered by Dr. Halley. Thus that mathematician, in
effect considered, that the Table of mortality was to be so far deemed
inviolate, that all subsequent deductions were to be framed in strict
demonstrative sequence from it, however laborious the relative calcu-
lations might appear.

De Moivre, on the contrary, although admitting the soundness of
this reasoning, *per se*, yet doubted whether a Table could be con-
structed worthy of this implicit confidence, either in fact, or as the
warrantable basis of long tedious computations; and that therefore,
any uniform theory offering a plausible and yet general interpretation
of the Law of Nature, was better worth reliance on the faith of its
generality, than could be awarded to the imperfect individualities of
data necessarily limited to the few years' observations then current.

Simpson's declaration was thus evidently more in confirmation of
Dr. Halley's, than of De Moivre's, notions, although, as we have
already anticipated, (vide p.   ,) the latter's publication supplied
Simpson with a very important practical formula wholly independent
of the peculiar nature of " De Moivre's hypothesis."

In some of the copies of the first, and throughout the subsequent
Edition of Simpson's work, De Moivre's name is wholly unmentioned;
in other copies the preface contains the following indirect acknow-
ledgement, of the occasional advantage Simpson had derived from
the perusal of that author's Treatise:—" Yet I would not be thought
to condemn any hypothesis grounded upon reason and matters of fact,
because such are oftentimes made use of to very great advantage,
of which Mr. De Moivre's excellent book on this subject is an
instance." (p. vi.)

he thought most eligible for adoption, Mr. Simpson observes:

" Of all that has hitherto been offered for estimating the probability
" of the duration of life, nothing seems deduced with greater judg-
" ment and exactness, than the tables publish'd by Dr. Halley, and
" Mr. Smart, for this purpose; which, nevertheless, are both liable
" to considerable objections.

" The Doctor's Table, being grounded on observations made at
" Breslau, a place where the generality of people live to a greater age
" than at London, (as appears by comparing the bills of mortality
" here, with those observations) can be no just measure of the proba-
" bility of life in this place; and as to that of Mr. Smart, tho' it is
" indeed free from this objection, and founded on a very large number
" of observations, yet the great and continual afflux of people from
" all parts up to town, renders the deductions from those observations
" considerably different, in one part of life, from what they would
" otherwise be; and this Mr. Smart seems not, in his table, to have
" considered, or made any allowance for.

" For these reasons, tho' I had determined to depend on, and make
" use of, this last gentleman's observations, in the ensuing pages (as,
" undoubtedly, the best for the city of London, and parts adjacent)
" yet have I deem'd it necessary to make some alterations, in the
" table of the probability of life, from thence derived.

" In doing this, I have supposed the number of persons coming to
" live in town, after 25 years of age, to be inconsiderable, with re-
" spect to the whole number of inhabitants; and therefore the proba-
" bilities of life, for all ages above 25 years, the same as this author
" has made them; but then have increased the numbers of the living,
" corresponding to all ages below 25; so that they may, as near as
" possible, be in the same proportion one to another, as they would
" be, were they to be deduced from observations on the mortality of
" those persons only, that are born within the bills. Which was
" done, by comparing together the number of christenings and
" burials, and observing, by help of Dr. Halley's table, the proportion
" which there is between the degrees of mortality at London and
" Breslau, in the other parts of life, where the ages are greater than
" 25." (Simpson Treat. Annuities, 1742, pp. 1-3.)

Mr. Smart's results having been inserted in a pre-
ceding page, it may be well to here append the al-

tered numbers as adjusted by Mr. Simpson, the re-
maining terms being common to both Tables:

Ages.	Number of Persons.	Decrement. *	Ages.	Number of Persons.	Decrement.
Born	1280		13	504	
		410			6
1	870		14	498	
		170			6
2	700		15	492	
		65			6
3	635		16	486	
		35			6
4	600		17	480	
		20			6
5	580		18	474	
		16			6
6	564		19	468	
		13			6
7	551		20	462	
		10			7
8	541		21	455	
		9			7
9	532		22	448	
		8			7
10	524		23	441	
		7			7
11	517		24	434	
		7			
12	510				
		6			

After explaining the fundamental properties of a
Table of Mortality, Mr. Simpson proceeds to exem-
plify some leading principles in the Doctrine of
Chances, and ultimately developes their application

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* The numbers representing " the decrement," or those who " die
off yearly," in this as in the former instance, have been, for per-
spicuity, inserted in a separate column.   Simpson appears to have
been the first to employ the word " decrement " (since generally
adopted) to express the tabular number of deaths; a relative interpre-
tation in strict analogy with the previous use of the term as opposed
to " increment " in the Doctrine of Fluxions.

by general formulæ, &c. to a variety of Life-contingency problems.

The enunciation and resolution of these form the principal portion of the treatise, though it also includes several popular examples and tables, founded on the London Table of Mortality, including one for Annuities on Single Lives, at 3, 4, and 5 per cent. interest, and four for ascertaining by approximation the value of annuities on two and three Lives.

In illustration of the various Problems, &c. Mr. Simpson originated the employment of a specific system of notation; by the aid of this he was enabled to present solutions which were not dependent upon numerical intrepretations, and thus the advantage was gained of the *doctrine* being kept wholly distinct from the various figural results it was capable of affording.

The avowed importance of this distinction, is the chief characteristic of Mr. Simpson's Treatise, and has been so fully acquiesced in by subsequent writers, as to have gradually created almost perfect concurrence as to the proper methods of calculation, although their opinions of the validity of particular tables of mortality, have essentially differed from time to time. Subsequent investigations have indeed revealed to us, that Simpson's *formulæ* were capable of improvement, but the principle of relying solely upon the doctrine of Mathematical "Probability," in demonstrating Life-contingency problems, without being restricted to any particular set of numerical results, is still happily recognized by calculators, and undoubtedly owes its prevalence to the pre-eminent example thus afforded by this celebrated Mathematician.

[1743] The publication of Simpson's work appears to have aroused the anger and jealousy of De Moivre, for in the following year (1743) he published a second edition of his Treatise on Annuities, which he described as " plainer, fuller, and more correct than the former;" allowing himself to be so far misguided by bitterness of spirit, as to add the following unmanly sentence to the Preface: " After the " pains I have taken to perfect this Second Edition, it may happen, " that a certain Person, whom I need not name, out of Compassion " to the Public, will publish a Second Edition of his Book on the " same Subject, which he will afford at a very moderate Price, not " regarding whether he mutilates my propositions, obscures what is " clear, makes a Shew of new Rules, and works by mine; in short, " confounds, in his usual way, every thing with a croud of useless " Symbols; if this be the case, I must forgive the indigent Author, " and his disappointed Bookseller." (De Moivre's Annuities, 1743, p. xii. Second Edit.)

In answer to this indirect allusion, Simpson, in the same year, issued an "Appendix, containing some remarks on a Late book on the same subject," in which he says, " It is not without some concern, that I think " myself obliged to take notice of the treatment of an author, of " whose judgment and candour I had conceived a very high opinion, " and whose name and writings I have always mentioned with respect. " It is not my design to expatiate on the unseemliness of this gen- " tleman's usage, nor to gratify a passion, which insinuations so gross " must naturally excite in a mind that looks with contempt on such " unfair proceedings; but only to offer a few particulars to the consi- " deration of the publick, with no other view than to clear myself " from a charge so highly injurious, and do justice to the foregoing " work." (Appendix, p. 1.)

The dispute between these eminent men need not be here further detailed, but the general reader will probably premise, that a subject which De Moivre and Simpson, the leading mathematicians of their

own time, thought worthy of dispute, must also, in the present day, when the doctrine of Life-contingencies is so prevalently studied, be no less entitled to rank as a topic of mathematical consideration.

It will be readily supposed from the hasty spirit in which De Moivre's Second Edition was issued, that it intrinsically offers but little extra comment on the present occasion, the insertion of three Tables of Life Annuities, at 4, 5, and 6 per cent., in accordance with his hypothesis, forming the chief difference between the Editions, the remaining alterations being principally those of re-adjustment or amplification.

[1744.] In the following year, (1744,) De Moivre published a paper in the Philos. Transactions, (No. 473, Vol. 43, p. 65,) to determine the value of Life-annuities, when a proportionate payment was allowed, according to the interval between the last payment received, and the date of death. He partly resolves this question in relation to Tables of Observations, with the extra supposition, that infinitesmal portions were to accrue for each moment that might intervene between the day of death and date of last payment; an assumption too theoretical to be further noticed on the present occasion.*

[1746.] In 1746, Mr. Hayes's work (vide p. 54,) was reprinted, thus indicating, that his labours, though merely of a popular character, had not been uselessly bestowed.

[1747.] In 1747, was published " The Calculation of " Annuities on Lives deduced from the London Bills of Mortality, &c. " by James Hodgson, F.R.S. &c."

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* The word infinitesmal has been merely used in the text as a popular interpretation of the fluxional increment employed by De Moivre.

Mr. Hodgson was well known as a mathematician, but his tract followed too closely upon the date of Simpson's to excite much attention.

His work is chiefly of a demonstrative character, in opposition to merely imaginary methods, but does not offer any particular topic for present notice, otherwise than to observe that he extended his Annuity-values to four places of decimals, whereas Dr. Halley and De Moivre merely computed theirs' to two, while Simpson's Tables presented but one.

[1750.] In 1750, appeared "New Observations, &c. on " City, Town, and Country Bills of Mortality, by Thomas Short, " M.D."

Dr. Short must have devoted considerable time and attention to collect all the results embodied in his work, consisting of multifarious data in connexion with burials, baptisms, &c. obtained by actual inspection of the registers in different parts of the country, and yielding a total of several thousand recorded deaths. The treatise, however, is so unskilfully arranged, as to be almost statistically unavailable; for though the most minute particulars are inserted in some respects, yet the ages are, in nearly all cases, omitted.

The following description of one of the tables will afford the general reader sufficient illustration of Dr. Short's well-intentioned industry:

" The Line below the yearly Totals, Column 1st, gives the Number " of rainy days that Year; Column 2d, how many People died on " these Days; Column 3d, how many Days were showry, besides the " rainy Days; Column 4th, the Number that died on those Days; " Column 5th, Number of drisling Days; Column 6th, how many " died; Column 7th, on how many Days Thunder was heard there; " Column 8th, how many died there; Column 9th, the Number of " Days on which it snowed; Column 10th, how many died." (p. 373.)

The table further comprises the specified number of deaths during the prevalence of the wind in "eight points" of the compass respectively, though the ages at which the deaths occurred, are not even incidentally mentioned.

From among the numerous general indications presented by Dr. Short, the two following tables are selected, as affording a comparison between the results of 26 of the most "sickly years," and those pertaining to an equal number of "healthy years."

26 Sickly Years.			26 Healthy Years.		
Year.	Baptized.	Buried.	Year.	Baptized.	Buried.
1618	7735	9614	1604	5458	5219
20	7845	9712	11	7014	7343
23	7945	11112	16	7985	8072
24	8299	12210	19	8127	8009
32	9584	9535	26	6701	5734
34	9855	10900	33	9997	8392
49	5825	10565	39	10150	9862
52	6128	12574	48	6544	9894
54	6620	13247	50	5612	8764
56	7050	13921	71	12510	15729
58	6170	14993	75	11775	17244
61	8855	19771	87	14951	21460
70	11997	20198	95	13876	19047
74	11851	21201	96	14861	18638
81	13355	23971	1700	14639	19443
93	13632	24100	2	15687	19481
1701	15616	24071	6	15369	19847
10	14828	24620	11	14706	19833
14	17495	26569	15	17234	26569
19	18413	28347	17	18475	23446
23	19203	29197	32	17788	23358
26	18808	29647	35	16873	23538
29	17060	29722	38	16060	25825
33	17465	29233	39	16181	25432
40	15231	30811	1673	11895	17504
41	14957	32169	1680	12747	21053
Totals	311922	522010	Totals	323415	418136
Medium	12000	20080	Medium	12439	16082

(Dr. Short's New Observations. 1750. p. 188.)

In this year also, (1750,) De Moivre published a Third Edition of his Treatise on Annuities; it was, however, merely a reprint of the former, with the exception that the sentence reflecting on Simpson's publication was omitted.

[1751.] In 1751 was published, " Observations on " the past growth and present state of the City of London. To " which are annexed, a Complete Table of the Christenings and " Burials, &c. by Corbyn Morris."

Mr. Morris's treatise is principally discursive, but contains several acute reflections on the subject of mortality, and some very judicious hints as to the better arrangement of the London Bills. The following quotation will illustrate, as compared with Mr. Smart's first recommendation of the insertion of ages, (vide p. 53,) that the subject was still engaging the attention of occasional writers.

" The present annual bills of mortality for London, though kept " in a better method than formerly, are yet defective in many parti- " culars of the most important concern.

" 1st. The extent of time at the beginning of life, is not suffi- " ciently divided, although the degrees of strength at this time are " continually varying from each other, and with swift steps.—Inso- " much, that one month in the earliest infancy produces greater alte- " rations than several years in manhood.—These successive and vio- " lent ravages in infancy ought therefore to be minutely described, in " order to furnish cautions for the better security of life in its tender " years; upon which the very preservation, and much more the " increase, of our species principally depends.

" 2dly. The diseases are not connected with the several ages: " thus, for instance, although it appears by the bill for the last year, " 1750, that 4543 died of consumptions, 5837 of convulsions, 4294 " of fevers, and 1229 of the small-pox; yet of what ages these sev- " eral persons were, is uncertain.—Whereas, by annexing to every " disease the number of persons of each age dying thereof, it would " appear, from remarks upon a competent series of years, to what " diseases persons of every age were particularly subject. From

" whence, if these bills were extended throughout the kingdom, the
" growth or declension of every particular disease would be clearly
" discovered.—This would furnish a constant fund of instruction to
" the intelligent physician;—and at particular critical junctures might
" suggest cautions, not unworthy the attention of the legislative
" body of the kingdom;—at least, would make a faithful report to it
" of the state of the national health, and of the annual increase of
" diminution of the people." (Corbyn Morris's Obs. 1751. Obs. III.)*

In the same year, (1751), a paper appeared in the Philos. Trans. (Vol. 47, p. 333,) on the " Improve-
" ment of the Bills of Mortality," by Mr. Dodson.

Mr. Dodson was principally known as Editor of the Mathematical Repository, the first volume of which was dedicated to De Moivre. In the paper above alluded to, Mr. Dodson strenuously recommended the Hypothesis of that author, as the most eligible means for determining the value of Annuities until the results derivable from the London Bills should be more worthy of confidence, by greater accuracy being observed in the method of collection. In aid of this, Mr. Dodson offered several suggestions as to their general improvement.

In this year also Mr. Weyman Lee published " A valuation of Annuities and Leases certain for a Single Life, &c." and attempted to refute the various charges of error, &c. which his former publication (vide p. 62,) had elicited.

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* Although the utility of Mr. Morris' suggestion is almost axiomatic, yet the specification of ages in relation to the various diseases or causes of death does not appear to have been published in the Official Statements until so late a period as August, 1842-3, (*vide* 4th Annual Report of the Registrar General, p. 194, *ibid* 5th Report, p. 134. Report on the Census (1841) of Ireland Appendix, p. 8.)

[1752] In 1752, Mr. Simpson published his "Select Exercises," and devoted the Sixth Part to "The valuation of Annuities for Single and Joint "Lives." The substance of this part was nearly the same as that of his Treatise formerly noticed, but his dispute with De Moivre had evidently occasioned Simpson to bestow more consideration upon the subject. Thus, in place of the former mixed table of mortality, of partly new numbers, and others partly derived from Mr. Smart's results, by which procedure, the table was made to commence at the irregular number 1280, the radix in this new publication was assumed as 1000, and the other terms accordingly recalculated. The tabular values, moreover, for Joint Lives, &c., in the former work had been merely specified for "mean" ages, whereas in the latter, the distinctive ages were inserted with the proper values, and a table of survivorship values added.

In this second treatise of Simpson, was inserted a new species of table, which, in principle, has subsequently been universally admitted into treatises on Life-contingencies, under the title of a Table of the Expectation of Life. The term expectation we have already seen was introduced by De Moivre to express the average duration of life, and by the peculiar nature of his theory, such average duration was exactly equal to half the number of years between the particular age and the "limit" of Life. (vide p. 50). When, however, De Moivre's hypothesis was disregarded, the expectation, or average duration of life, ceased to be ascertainable by any immediate rule, and, thus each value had to be separately calculated. Simpson appears to have thought

it well to supply this desideratum, and, accordingly tabulated the results for each age; thus originating the first Table of the Expectation of Life. (*vide* Simpson's Select Exer. 1752, p. 255.)

In the same year (1752,) De Moivre published a Fourth Edition of his Treatise on Annuities. The substance of this Edition merely differed from that of the second and third, by containing two extra Tables for Single Life-Annuities, at the rates of 3 and $3\frac{1}{2}$ Interest, and some directions as to the employment of Interest Tables in general. As a species of Note to Problem XXIII, De Moivre added the following remark, which indirectly pourtrays, that the subject of " Life-Assurance " was beginning to attract attention as a matter of calculation, though merely in connection with annuity transactions.

" By this Proposition, some useful questions concerning Insurances
" may be resolved.

" Suppose A, at 30 Years of Age, assigns over to B an Annuity
" of 1000*l*. a Year, limited to 10 Years, and depending likewise
" upon A's Life: then, by the foregoing Solution, A ought to receive
" for it only 7018*l*. 8*s*. Interest being at 5 per Cent. But if B
" wants that the Annuity should stand clear of all Risques, he must
" pay for it the Value certain, which is 7721*l*. 4*s*. and A ought to
" have his Life insured for 702*l*. 16*s*. the just Price of such an In-
" surance being the Difference of the Values of the Annuity certain,
" and of the same Annuity subject to the Contingency of the Annui-
" tant's Life failing." (De Moivre's Treatise on Annuities, 4th Edit.
1752, p 77.)*

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* It may be well to observe that during the last century the term
" Life," in works on Annuities, &c. was frequently used to express
Life-Annuity, thus, the " value of a Life," signified the value of a
Life-Annuity or Lease. It is evidently in this sense that De Moivre,
in the above quotation, states, " A ought to have his Life insured
for £702 16," *i. e.* £702 16, would insure to B the completion of

[1753] In 1753, Mr. Dodson, as Editor, published a second volume of the Mathematical Repository, and included a numerous collection of Annuity-questions, resolved in accordance with De Moivre's Hypothesis. Mr. Dodson moreover calculated to three places of decimals, a complete set of Annuity Tables for Single Lives, at the several rates of 3, $3\frac{1}{2}$, 4, $4\frac{1}{2}$, 5 and 6 per Cent. Interest, thus incidentally offering the fullest embodiment of De Moivre's Hypothesis extant.

[1754] In the following year (1754) Mr. Dodson contributed a paper to the Philos. Transact. (vol. 48, p. 487) in which he refers to De Moivre's Calculation of the part-Annuity payment in relation to the exact date of death, and offers a method of determining the value without resorting to the employment of infinitesimal quantities. (*vide* p. 74).

In the same year was published " The Valuation of Annuities on Lives, deduced from the London Bills of Mortality, &c." by S. Stonehouse. Mr. Stonehouse's Treatise is merely of a popular character, and nearly similar in purpose to the superior works of Simpson and Hodgson, already mentioned.

[1755] In the preceding page De Moivre's observation as to "Insurance," was noticed as indicating that the subject of *calculative* Life Assurance was exciting observation. Mr. Dodson's attention seems to have been attracted by the *new* topic, for

the Annuity if A should die within the period of ten years. This is further evidenced by the remark which De Moivre subjoins, " The same £702 16 is likewise the value of the reversion of this Annuity to a Person and his Heirs who should succeed to the remainder of the ten years, upon A's decease." (*ibid.*)

in 1755, he published a third volume of the " Mathematical Repository," " relating to Annuities, Reversions, Survivorships, Insurances, and Leases dependent upon Lives, in which it has been endeavoured to exhaust the subject."

In his preface, after excusing the various observations he had adopted throughout the work, " in order to prevent the swelling thereof beyond a proper size," he adds that no apology would be required for the introduction of the calculations " concerning Insurances on Lives, as it is a subject not before handled, and will shew its own use." (Dodson's Mathematical Repository, 1755, vol. III, p. iv.)

He only, however, devotes about five of 372 pages to so important a subject, and limits his illustrations to the three following questions, which appear to constitute the first directly specified Life-Assurance cases upon record.

The first question proposed by Mr. Dodson is, " What sum ought to be paid for insuring for one year a given sum s, " on a life whose complement is n ?"*   Example—If the sum of £100 " is Insured on a Life aged sixty-six years: then " £5 will be the " required premium."†

The second question is, " What constant yearly sum, of " which the first payment is to be made immediately, ought to be " paid for insuring the sum s, on a Life whose complement is n?" Example—What constant annual premium should be paid for insuring

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* The term, Complement, as applied to the duration of Life, is derived from De Moivre's Treatise, " I call that the Complement of " Life, which is the time comprehended between an Age given and " the extremity of old age.   Thus, if the age be 50, and the extre- " mity of old age be 86, the Complement of Life is 36." (De Moivre on Annuities, 1st Edit. 1725, p. 75.)

† Mr. Dodson solves this question by virtually equating the premium (modern notation) to $1 - {}^{1}aa^{-1}$, thus disregarding the discounted value.

£100 on a Life aged sixty-six, allowing 4 ⅌ cent. compound interest? "8,481" (8.9.8) "will be the annual premium required."*

The last question proposed by Mr. Dodson is, "A, "who has a place, or other annual income for life, would borrow a "sum of money of B, and (besides a bond, bearing interest) would "insure his life, as a collateral security; B undertakes to be the "insurer, and to remit the payment of the principal, to the heirs of "A, if he constantly pays the said interest and insurance during his "life to him, B, or his heirs; what sum ought A to pay, annually "for interest and insurance, for £1 so lent?" Example—If A, the borrower, be aged sixty-six, and interest be computed at 4 ⅌ cent. then, "0,125" (2s. 6d.) "will be the sum required." (Dodson's Mathematical Repository, 1755, Vol. III, p. 365.)†

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* Mr. Dodson resolves this question by the equation (modern notation)

$$p = (1 + A)^{-1}(\mathfrak{A} + {}_1 \, av \, \underset{1|}{\overset{1}{\mathfrak{A}}} + {}_2 \, a \, v \, \underset{1|}{\overset{2 \, 2}{\mathfrak{A}}} \text{ seriatim ad } \underset{n}{\overset{n \, n}{a \, v}} \underset{1|}{\mathfrak{A}})$$

an expression by which in principle "ascending scales," in the present day, are demonstrated. He invalidates, however, his own result, by disregarding, as in the former case, the first year's discount. This misapplication he appears to have overlooked in the present instance, in consequence of the total result incidentally coinciding with that, of the then current, though erroneous formula,

$\mathfrak{A} = \dfrac{P - A}{P}$ (in which $P = \frac{1}{r}$) in place of the correct expression,

$\mathfrak{A} = \dfrac{P - A}{P + 1}$, as afterwards first established by Dr. Price. (*vide* Obs. Rever. Pays. 1st Edit. 1771, p. 31.)

† This was solved by Mr. Dodson, to the effect that $\dfrac{1 + r}{1 + A}$ was the required annual charge, but as he, in a scholium, deducts the premium from the £1 to be lent, and had previously equated the premium though falsely, to $\dfrac{1 - rA}{1 + A}$, so the borrower in respect of $1 - \dfrac{1 - rA}{1 + A}$,

was granting an annual charge of $\dfrac{1 + r}{1 + A}$. Now $1 - \dfrac{1 - rA}{1 + A}$ is in ratio to $\dfrac{1 + r}{1 + A}$ as 1 is to $\dfrac{1}{A}$. A result consistent with modern calculations, $\left(\dfrac{r + p(1 + r)}{1 - p(1 + r)} \right)$ (*vide* Jones' Treat. Ann. Art. 244) the faulty determination of the premium having been indirectly nullified by the reconversion in the scholium.

The general reader will thus perceive that the subject of Life-Assurance primarily hinted at by De Moivre, and re-considered by Mr. Dodson, was at length pourtrayed as a distinct object of calculation, without being deemed necessarily conjoined with Annuity transactions.

In this, as in the other volumes by Mr. Dodson, De Moivre's Hypothesis is chiefly employed, and by the aid of it, Mr. Dodson solves questions extending to four Lives, and moreover institutes a notation and method of arranging probabilities, which have been since more or less adopted in principle, although Mr. Dodson's formulæ being solely theoretical, have long since become obsolete.*

In the same year (1755) Dr. Brakenridge, in a paper inserted in the Philosophical transactions, proposed that as the Bills of Mortality were supposed to include the deaths of many persons merely temporarily resident in London, with other causes of excessive indications of relative mortality compared with the population generally, and as the Breslaw Observations were thought to be too favourable, so a new Table might be usefully formed, by combining the two sets of data, and thus correct the evident disproportions occasionally displayed by the London Bills. In pursuance of this idea, he accordingly

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* If the reader, generally conversant with such matters, will consult Mr. Dodson's Mathematical Repository, Vol. III, pp. 108-205, &c. the mode of arrangement alluded to, will be recognised as the original of several methods still in use in mixed contingencies, particularly those admirably pourtrayed by Mr. Milne, in his Treatise on Annuities, vol. I, p. 191, *et seq.*

offered from the conjoint materials, a regulated Table of Mortality, which the reader will find set forth in Vol. 49, p. 182, of the publication referred to.

[1756] In 1756, was published, the Third Edition of De Moivre's " Doctrine of Chances," including the usual species of Appendix on Life-Annuities, &c. with a few miscellanea added. Among these were the four Tables of Mortality then generally known; two of these have already been brought under the notice of the reader, viz. Dr. Halley's, (1693) and Messrs. Smart's and Simpson's, (1738-1742); the remaining two were proposed by foreign authors, one by M. Kersseboom, (1738), representing the Mortality in Holland, among the Dutch Annuitants; the other, by M. De Parcieux, (1746), derived from the List of claimants for the French Tontines. De Moivre remarks on each of these, and in illustration of Dr. Halley's says, " We may therefore retain this last as no " bad standard for mankind in general; till a better Police, in this " and other nations, shall furnish the proper data for correcting it, " and for expressing the decrements of Life more accurately, and in " larger numbers.

" For which purpose, the parish Registers ought to be kept in a " better manner, according to one or other of the Forms that have " been proposed by Authors. Or, if we suppose the numbers an- " nually born to have been nearly the same for an age past, the thing " may be done at once, by taking the numbers of the living, with " their ages, throughout every Parish in the Kingdom: as was in part " ordered some time ago by the Right Reverend the Bishops: but " their Order was not universally obeyed; for what reason we pretend " not to guess. Certain it is, that a Census of this kind once esta- " blished, and repeated at proper intervals, would furnish to our " Governours, and to ourselves, much important instruction of which " we are now in a great measure destitute: Especially if the whole " was distributed into the proper Classes of married and unmarried, " industrious and chargeable Poor, Artificers of every kind, Manu-

" facturers, &c. and if this was done in each County, City, and
" Borough, separately; that particular useful conclusions might
" thence be readily deduced; as well as the general state of the
" Nation discovered; and the rate according to which human Life is
" wasting from year to year." (De Moivre's Doctrine of Chances,
1756, 3rd Edit p. 347.)*

In this year also, (1756) Mr. Dodson contributed
to the Philos. Transact. (Vol. 49, p. 891,) a Table of
Annuity-values based upon the compound Table of
Mortality, constructed by Dr. Brakenridge, from the
Breslaw and London Bills, as already noticed.

[1759] In 1759 was published anonymously a
" Collection of the yearly Bills of Mortality, from 1657 to 1758 in-
" clusive, together with several other Bills of an earlier date."
The treatise also contains a reprint of Graunt's,
Petty's and Corbyn Morris' publications, with
" A Comparative view of the Diseases and Ages, and a Table of the
" probabilities of Life, for the last thirty years, by J. P. Esq. F.R.S."

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* The establishment of a Census, so strenuously urged by De
Moivre, and then recently instituted in Sweden, (1749, 52, 55) (vide
Trans. Stockholm Academy or Keralio's Abridg.) was not, however,
practically adopted in England until 1801, (vide 41 Geo. III (G. B.)
c. 15.) The process was repeated at the end of the decennial in-
terval (1811), but it was not until 1821 that the ages of the com-
munity were specified. This really useful distinction was unfortu-
nately considered superfluous in 1831, (vide Parl. Rep. Sess. 1830,
No. 385-460,) and though it has been rigorously adopted in 1841,
yet the method of collecting the enumerative details is necessarily so
different from that employed twenty years previously, that Actuaries
have hitherto derived but little corroborative aid from this otherwise
important decennial regulation. That the validity, however of our
national statistical returns is daily increasing, and, by the increased
stimulus, will eventually amply repay the vital statiscian for the pre-
vious desultory method of proceeding, is fully pre-indicated by the
elaborate character of the last Census (1841), and the excellent
Reports and Appendices annually issued from the General Register
Office.

It is stated in the Encyclopædia Britannica, seventh edition, part 57, p. 515, that the treatise above alluded to was arranged by Dr. Birch, Secretary of the Royal Society, from materials furnished by Dr. Heberden ; and that the Table of Probabilities was calculated by James Postlethwayte, Esq.

The publication is, in every respect, worthy of being consulted, especially by statiscians interested in the early history of the London Bills of Mortality, these being printed at full length, and a very judicious preface prefixed. It may be well, however, to add, that certain of the London Bills having been frequently copied by different printers, the compilers of the above collection do not seem to have always procured the primary issues, and have thus occasionally reprinted in the totals, &c. typographical errors which did not exist in the original copies.

⌈1760.⌋ In the present Essay, there has hitherto been occasion to notice but four Societies; " the Amicable," (p. 35) " the Union," " the Royal Exchange," and " the London," (p. 42) by each of which, distinctions as to the ages of applicants were disregarded. During the succeeding forty years (1720-60), the reader is already aware the doctrine of Life-contingencies had been illustrated by numerous authors, and the subject of distinction as to age prominently exemplified, from date to date.

Accordingly, in 1760, an application was made to the Crown, for a charter to incorporate a *new* Life Assurance Society, to be founded upon specific calculations, by which the premiums were to be regulated, and made to vary according to *age*, in pursuance of recommendations repeatedly urged by

Mr. Simpson, in his mathematical lectures, and practically exemplified by Mr. Dodson, in several calculations prepared for the purpose.

[1761] The application for a charter was referred to the Attorney and Solicitor General who, in answer to the petition, framed the following report, which is here inserted entire, not only as collaterally serving to exemplify the nature of the petition itself, but also from the application necessarily constituting an incident of too prominent a character in the history of the subject, to warrant curtailment in the present instance.

" In humble Obedience to your Lordships' Order of the 2nd Day
" of April, 1760, referring to us the annexed Copy of a Petition of
" the Hon. Coote Molesworth, of Chichester, Doctor in Physic, and
" Fellow of the Royal Society; Sir Richard Glynn, Knt, and Bart.
" Alderman of London; Thomas Pickering, of London, D. D.; John
" Silvester, of London, M. D., F. R. S.; and 78 others, in Behalf of
" themselves and many others, His Majesty's dutiful and loyal Sub-
" jects, for our Consideration, and to report our Opinion thereupon to
" your Lordships, which Petition sets forth,

" That great Numbers of His Majesty's Subjects, whose Subsist-
" ence principally depends on the Salaries, Stipends, and other In-
" comes payable to them, during their natural Lives, or on the Profits
" arising from their several Trades, Occupation, Labour, and Indus-
" try, are very desirous of entering into a Society for insuring the
" Lives of each other, in order to extend after their Decease, the
" Benefit of their present Incomes to their Families and Relations,
" who may otherwise be reduced to extreme Poverty and Distress, by
" the premature Death of their several Husbands, Fathers, and Friends,
" which humane Intention the Petitioners humbly apprehend cannot
" be effectually carried into Execution without His Majesty's Royal
" Authority to incorporate them for that Purpose.

" That to effect their said humane Intention, the Petitioners beg
" leave to propose the following Plan, viz.—

" That the Petitioners shall form themselves into a Society for the

" Assurance of Lives, and that they and their Successors shall have
" Power to grant Policies for that Purpose.

" That on granting such Policies, the Petitioners and their Succes-
" sors shall receive, and from Time to Time continue to receive, from
" the Persons to whom the same are respectively granted, a Premium
" or Premiums proportionate to the Chance of Death attending the
" *Age* of the Life or Lives assured, and the Term of Years for which
" the same shall be respectively assured.

" That on granting such Policies, the Petitioners and their Succes-
" sors shall also receive from the Persons to whom the same are
" respectfully granted, a Deposit of a further Sum to answer the Ends
" of a joint Stock or Fund; which Deposit shall be placed out on
" Government or other sufficient Securities, and there remain to make
" good any Deficiency or Deficiences that shall or may happen to be
" in the Fund arising from Premiums, by Means of an unusual Mor-
" tality amongst the Members of the said Society, or by any other
" means whatsoever.

" That no such Policy or Policies of Assurance shall be granted
" to any Person or Persons until he or they shall have signed or
" executed a Declaration or Covenant, purporting that he or they do
" voluntarily enter into and become a Member or Members of the
" said Society, and will so continue during the Term or Terms for
" which the Policy or Policies to be granted to him or them shall
" respectfully continue in Force, and will bear or pay their Proportion
" or Proportions of any Loss or Losses, which, during such Term or
" Terms, shall or may happen to the said Society, and exceed the
" Amount of the Premiums and Deposits to be paid as aforesaid.

" That the Petitioners are ready and desirous to ascertain and fix
" the several Premiums and Deposits proposed to be by them and
" their Successors taken as aforesaid, and also to be restrained from
" acting contrary to their said Plan.

" That Establishment by His Majesty's Royal Charter of a free and
" open Office of Insurance, upon the Plan aforesaid, will, as the Peti-
" tioners, with great submission, apprehend, be more equitable than
" any hitherto proposed, as being calculated for the sole Benefit of
" the Persons assured, a method not hitherto practised; and will, as
" the Petitioners humbly hope, in a Variety of Instances, prevent the
" before-mentioned Inconveniences, and be productive of the greatest
" Advantages to the Public.

" That there is at present subsisting but one Corporation for per-

" petual Assurance on Lives, which, as the Petitioners humbly appre-
" hend, acts upon so circumscribed and narrow a Plan, that very few
" of His Majesty's Subjects do receive any Benefit from it, in Com-
" parison of the great Number to whom the Benefit of such an
" Insurance might be extended.

" The Petitioners therefore humbly pray that His Majesty, out of
" his Royal Grace and Favour, will be pleased to grant His Majesty's
" Royal Charter for the Purposes aforesaid, unto   *   *   *
" by the Name of ' The Corporation for Equitable Assurances on
" Lives," under such Restrictions as to His Majesty in his Royal
" Wisdom shall seem meet.

" Upon this Petition your Lordships will be pleased to observe,

" 1st. The Petitioners propose to insure upon cheaper Terms, and
" for a longer Time, than is practised at present in any Offices, to
" which End they have specified the Rates at which the Assurance is
" to be done.

" 2nd. They propose to raise a Capital, by investing the Premiums,
" together with a small additional Sum of 40s. to be deposited by
" every Person insured, to answer all Losses; and by way of further
" Security, to oblige every Person insured to become a Member of
" the Corporation, and to declare or covenant that he will bear his
" Proportion upon any Call, if the Premiums and Deposits should
" prove deficient.

" Upon these Proposals we are required to deliver our Opinions,
" whether it will be proper for His Majesty to grant the Petitioners a
" Charter of Incorporation for these Purposes.

" We having been attended with Counsel on Behalf of the said
" Petitioners, and also by Counsel on Behalf of the Governors and
" Companies of the London and Royal Exchange Assurance Com-
" panies, and also on the Behalf of the Corporation of the Amicable
" Society, for a perpetual Assurance on Lives, in Serjeant's Inn; the
" said Companies and Corporations having entered Caveats with the
" Attorney-General against granting the Prayer of the said Petition,
" and the said Petitioners and their Opponents having produced the
" several Affidavits annexed to this our Report, we have proceeded to
" examine the same, and after the best Consideration we have been
" able to give the Subject, we are humbly of Opinion to advise His
" Majesty not to comply with the Prayer of this Petition for the
" following Reasons:

" 1st. Because it appears to us altogether uncertain whether this

" Project will or can succeed in the manner in which it is proposed;
" and if the Success is uncertain, the Fund for supporting it, which
" is to arise from the Profits of the Undertaking, will be precarious.

" This last Consideration is in our Opinion a fatal Objection to
" the Scheme, for though an Undertaking plainly calculated for the
" Benefit of the Public, may in some Instances deserve Encourage-
" ment, even where the Success is dubious, yet in such Cases, the
" Projectors alone ought generally to abide the Peril of the Mis-
" carriage.

" In the present Proposal therefore, whatever else may be hazard-
" ous, the Capital or Fund to answer Losses ought to be certain and
" liable to no Casualty, for which reason, when the Legislature enabled
" His Majesty to erect the two Corporations of the Royal Exchange
" and the London Assurance, they thought it necessary to oblige
" these Bodies, in the first place, to raise a large Capital before they
" began to insure.

" *The success of this Scheme must depend upon the truth of certain*
" *Calculations taken upon Tables of Life and Death, whereby the Chance*
" *of Mortality is attempted to be reduced to a certain Standard: this is*
" *a mere speculation, never yet tried in Practice, and consequently sub-*
" *ject, like all other experiments, to various chances in the Execution.*

" The Tables upon which the Calculations are built, are the Bills
" of Mortality of London, and the Breslau Tables, and admitting
" them to be strictly accurate (of which there is strong reason to
" believe the contrary) they are compounded of diseased as well as
" healthy Persons, of those who are embarked in dangerous as well
" as other Employments, without pointing out the Proportions they
" bear to each other, and yet as the Petitioners propose to ensure
" only such even of the healthy, as are not employed in dangerous
" Occupations, the Register of Life and Death ought to be confined,
" if possible, for the sake of Exactness, to such Persons only as are
" the Objects of Insurance; whereas the Calculations offered embrace
" the Chance of Life in general, the healthy as well as unhealthy
" Parts thereof, which, together with the Nature of such Persons'
" Occupations, are unknown Numbers.

" As the Fund to answer Losses must depend principally upon the
" Premiums (for we pay but little regard to the small Deposits or the
" personal Covenant,) the project should be sure of success; other-
" wise the Adventurers will be undone, or greatly injured, and the

" Calamity will fall the heavier, because it will fall principally upon
" the poorest Sort, the Rich having no temptation to insure. Under
" these circumstances, if there was no other Objection to the Scheme
" proposed, the Uncertainty of Success would make us fearful of
" advising the Charter.

" We are the more apt to doubt of the Event, because it has been
" represented to us by the Affidavit of Mr. Savage, that all the Profit
" which has been received by the Royal Exchange Assurance, from
" the time of its commencement to the present time, amounts only
" to a sum of £2,651 4s. 6d. the difference between £10,915 2s. 2d.
" paid in Premiums, and the sum of £8,263 17s. 8d. disbursed in
" Losses, which small Profit must have been near exhausted in the
" Charges of Management. If then this Corporation, who are
" charged with taking unreasonable Premiums, have reaped no greater
" Profit, we can hardly expect a more considerable Capital to arise
" from lower Premiums; and the hazard of Loss will be increased in
" Proportion as the Dealing will be more extensive.

" The Crown has very wisely been always cautious of Incorporating
" Traders, because such Bodies will either grow too great, and by
" overwhelming Individuals, become Monopolies; or else, by failing
" will involve thousands in the ruin attendant upon a Corporate
" Bankruptcy. As Trade seldom requires the Aid of such Combina-
" tions, but thrives better when left open to the free Speculation of
" private men, such Measures are only the Expedient where the Trade·
" is impracticable upon any other than a joint Stock, as was thought
" to be the case in the East India, South Sea, Hudson's Bay, Herring
" Fishery, and in some other Companies erected upon that Principle;
" but there does not appear to be any such Necessity in the present
" Case, because the Business of insuring Lives is carried on not only
" by the two great Companies already mentioned, but such Policies
" are duly underwritten by numbers of private men; and we think
" that, if the Profit was so enormous as the Petitioners have endea-
" voured to represent, upon the terms now, and for many years prac-
" tised in the City of London, there would not have been wanting
" enterprising Persons to have reduced the Premiums, and drawn this
" Branch of Dealing to themselves by underselling the Market. If
" the Petitioners, then, are so sure of Success, there is an easy
" method of making the Experiment, by entering into a voluntary
" Partnership, of which there are several Instances now subsisting in

" this Business of Insuring,* and, if upon such a Trial these Calcu-
" lations are found to stand the Test of practical Experiment, the
" Petitioners will then apply with a much better Grace for a Charter
" that they can at present, whilst the Scheme is built, only upon
" speculative Calculations.

" 3d. The Parliament, in erecting the two great Companies already
" mentioned, have sufficiently declared their Opinion, that such
" Charters ought not to be granted without some Benefit accruing to
" the Public, and were not sure when they passed the Act whether
" they were not erecting a Nuisance; to prevent which a Power was
" reserved to the Crown to abolish the Corporation at any time
" within the term of 31 years, if they should be found upon trial to
" be mischievous or inconvenient; and we cannot help observing that,
" except only in the Case of the Amicable Society, of Serjeant's Inn,
" and which is formed upon a very narrow Bottom, the Crown has
" never, of itself, so far as appeared to us, granted such a Charter as
" the present, in any Case whatsoever; and, as the two great Com-
" panies paid a very large sum to the Public for the Privilege of their
" Charter, we cannot advise the Crown to entrench upon their Rights
" on the bare Request of any set of men, without a clearer and more
" certain Prospect of public Good.

" It is for these Reasons principally that we find ourselves under a
" necessity of advising His Majesty not to comply with the Prayer of
" this Petition, and though we are fully persuaded that those worthy
" Gentlemen who have made this Application are really convinced
" that this Scheme will prove advantageous to the Public, as well as
" profitable to themselves, yet we have not sufficient Evidence to
" satisfy us that either of these Ends will be answered, or that any
" Necessity of the Times requires the Table to be extended under a
" Charter of Incorporation.

<div align="right">

" All which is humbly submitted to your
" Lordships consideration by
" Your Lordships most obedient Servants,

</div>

" Tuesday,   " C. PRATT,
" July 14th, 1761.   " C. YORKE."

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* The character of such partnerships is indirectly illustrated by
the preceding expression in p. 91, " The Chance of Mortality is at-
" tempted to be reduced to a certain standard, this is a mere speculation
" never yet tried in practice."

[1762.] The acquisition of a charter being thus rendered hopeless, the petitioners constituted themselves, by Deed of Settlement, (dated Sept. 7, 1762,) into a Society, by the name of " The Society for " Equitable Assurances on Lives and Survivorships."

The history of this justly-celebrated Establishment, is not only too commonly known to need detail, but having been made the subject of an express Treatise as lately as 1828, under the title of " A view of the Rise and Progress of the Equitable Society," by the Actuary of the Institution, (the late Mr. W. Morgan,) it only remains to direct the reader's attention to Mr. Morgan's publication, as the legitimate source, if further information be sought on this particular topic.

It will be readily surmised from the existing prevalence of Life-assurance transactions, that the intermediate period of eighty-two years, (1762-1844) has not passed without exciting considerable attention to the subject, and giving rise to numerous excellent Treatises and Societies; but the relative details are of so distinct a character from those of the earlier history, as to have induced the writer to consider the date of establishment of the first Life-assurance Society in which ages were distinguished, as the most suitable limit on the present occasion.

1693 $_na = {}^naa^{-1}$ (Deduced from Dr. Halley's "Estimate," Phil. Tr. vol. 17, pp. 596-610.)

$_n\mathfrak{a} = (a - {}^na)\,a^{-1}$

$A = a^{-1}\,({}^1a\,(1+r)^{-1} + {}^2a\,(1+r)^{-2} \text{———} \cdot \text{———} {}^na\,(1+r)^{-n}\,)$

$_n(ab) = {}^n(ab)\,(ab)^{-1}$

$_n(\overline{ab}) = [ab - ({}^na - a)\,({}^nb - b)]\,(ab)^{-1}$

$_n(\mathfrak{a}\mathfrak{b}) = (a - {}^na)\,(b - {}^nb)\,(ab)^{-1}$

$_n(\mathfrak{a}b)\,\&c. = (a - {}^na)\,{}^nb\,(ab)^{-1}$ &c.

$ABC = (abc)^{-1}\,({}^1(abc)\,(1+r)^{-1} \text{———————} {}^n(abc)\,(1+r)^n)$

1725 $A = {}^1aa^{-1}\,(1+r)^{-1}\,(1 + {}^1A)$ (Deduced from De Moivre's "Annuities on Lives," 1st. Edit. p. 6.)

$_{\neg|}{}^nA = {}^naa^{-1}\,(1+r)^{-n}\,{}^n_{\neg|}A$ (ibid p. 26.)

$\overline{AB} = A + B - AB$ (ibid p. 36)

$\overline{ABC}\,\&c. = A + B + C - (AB + AC + BC) + ABC$ &c. (ibid p. 42.)

$_{\neg|}{}^B A\,\&c. = \overline{AB} - B$ &c. (ibid p. 49) $= A - AB$

$a = a^{-1}\,(a + {}^1a + {}^2a \text{———————} {}^na - \frac{a}{2}\,)$ (ibid p. 82)

$S\,_n(\mathfrak{a}b)\,\&c. = S\,\frac{1}{2ab}\,({}^na - {}^{n+1}a)\,({}^nb + {}^{n+1}b)\,\&c.$ (ibid p. 96) $= \frac{1}{2}S\left[\,_n(\mathfrak{a}b) + \frac{n(a_1b)}{1a_1} - \frac{n(ab_1)}{1b_1}\right]$

1738 $_{\neg|}\mathfrak{A} = {}_1\mathfrak{a}\,(1+r)^{-1} + {}_2\mathfrak{a}\,(1+r)^{-2} \text{———————} {}_n\mathfrak{a}\,(1+r)^{-n}$ (Deduced from De Moivre's "Doctrine of Chances," 2nd Edit. p. 212.)

1742 $ABC\,\&c. = {}^1(abc)\,(abc)^{-1}\,(1+r)^{-1}\,(1 + {}^1(ABC)\,\&c.$ (Deduced from "Simpson's Doctrine of Annuities, &c." p. 18.)

$\overset{m}{\overline{ABC..n}} = (ABC..n)\left\{1 - n + \frac{n(n-1)}{1.2} - \frac{n(n-1)(n-2)}{1.2.3} + \frac{n(n-1)(n-2)(n-3)}{1.2.3.4}\&c. +\right.$

$(S \cdot C)[ABC..(n-1)]\left\{1 - (n-1) + \frac{(n-1)(n-2)}{1.2} - \frac{(n-1)(n-2)(n-3)}{1.2.3}\&c. +\right.$

$(S.C)[ABC..(n-2)]\left\{1 - (n-2) + \frac{(n-2)(n-3)}{1.2}\&c.\&c.\right.$ (ibid p. 26.) $= \overset{1}{o} - m\overset{2}{o} + \frac{m(m+1)}{1.2}\overset{3}{o} - \frac{m(m+1)(m+2)}{1.2.3}\overset{4}{o}\&c.$

A's Tontine Interest in $\overline{ABC..n} = A - \frac{A\overset{1}{O}}{2} + \frac{A\overset{2}{O}}{3} - \frac{A\overset{3}{O}}{4}\&c.$ (ibid p. 75.)

$A(h.y) = $ approx. $(A + \cdot 25)$ (ibid p. 79.)

$A(q^y.) = $ approx. $(A + \cdot 375)$ (ibid p. 79.)

1752 $ABC = $ approx. (AD) in which $D = BC$ (ibid Simpson's "Select Exercises," p. 279)†

1755 $\mathfrak{A} = {}_{\neg|}\mathfrak{A} + {}_1av\,{}^1_{\neg|}\mathfrak{A} + {}_2av^2\,{}^2_{\neg|}\mathfrak{A}\,\&c.$ (Deduced from "Dodson's Mathem. Repos�y." vol. III, p. 364.)

The two following formulæ having been incidentally noticed in the preceding pages, are also here included.

1771 $\mathfrak{A} = \frac{(\frac{1}{r} - A)r}{1+r}$ (p. 31.) $= \frac{\frac{1}{r} - A}{\frac{1}{r} + 1}$ (Deduced from Dr. Price's "Obs. on Rever. Payments." 1st Edit.)

1779 $^nA = S\,(_{n+1}av^{n+1} + {}_{n+2}av^{n+2} + {}_{n+3}av^{n+3}\&c.)\,(_nav^n)^{-1}$ p. 61. (Deduced from Morgan's "Doctrine of Annuities," 1st Edit.)

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* In thus attempting to arrange Life-contingency formulæ in their order of origination, it has been found necessary, to prevent inordinate extent, to only include those that are still existent; the primary *forms* being preserved as nearly as the symbols of modern notation will admit of. Among the notations currently known, Mr. Milne's appearing the best adapted for purposes of general arrangement, the writer has expressed the respective formulæ accordingly.

† The reader may be surprised to find so few formulæ attributed to Simpson, but it should be remembered that the present list includes only those that are still existent, consequently the compound formulæ in which Simpson committed the oversight of indirectly relying on $\mathfrak{A} = 1 - rA$ have necessarily been omitted. In other respects, we owe nearly the whole Doctrine of *Successive Lives* to the publications of this eminent mathematician.